THE GILBERT ARITHMETICS

BOOK THREE

GILBERT
AND
GILBERT

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BY

CHARLES H. GLEASON

PRINCIPAL OF THE SUMMER AVENUE SCHOOL
NEWARK, NEW JERSEY

AND

CHARLES B. GILBERT

FORMERLY SUPERINTENDENT OF SCHOOLS, ST. PAUL
NEWARK, AND ROCHESTER
AUTHOR OF "STEPPING STONES TO LITERATURE," "GUIDE BOOKS TO ENGLISH," "THE SCHOOL AND ITS LIFE," ETC.

BOOK THREE

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PREFACE

THE two main functions of an elementary arithmetic are to secure such familiarity with the four tables that the combinations become automatic, and to create inductively mathematical concepts and apperceiving bases. The work necessary to this latter end is expansive and suggestive rather than definitive.

Following this comes the need for a technical book which, while still suggestive, is increasingly conclusive, giving the children a satisfying sense of definiteness and mathematical certainty, — the only certainty possible to them. Such a book should also at least hint at the higher and broader uses of the subject.

Book III of the Gilbert Arithmetics aims to satisfy these demands of maturer minds. While taking for granted a knowledge of the tables and processes of the four fundamental principles, it contains a restatement in a more exact form of the essential definitions and rules given in Book II.

It includes a review and an amplification of denominate numbers and of common and decimal fractions.

The principles of percentage are fully demonstrated and traced to their origin in a manner so clear as to remove much of the difficulty usually attending the study of this subject.

The applications of percentage to the practical problems of ordinary business are treated very fully and simply, the chapter on banking and money matters being especially instructive.

The study of dimension is kept constantly before the student by the introduction of numerous problems in measurement throughout the book, and is given very full and definite treatment in the chapter on mensuration.

A chapter is devoted to the principles of algebra, including the use of letters to indicate quantities and of the equation in solving problems. Mathematically speaking, this is of the greatest consequence, introducing the child, as it does, to the higher forms of thinking involved in the use of general terms as distinguished from the merely specific.

An arithmetical statement of fact, though a generalization, stands for itself alone; $2 + 3 = 5$ remains that and nothing more, while $a + b = c$ stands for an infinite number of possible terms, for all of which it is equally true.

Dr. Oliver Wendell Holmes, in the *Autocrat*, wittily puts it thus:

"I was just going to say when I was interrupted, that one of the many ways of classifying minds is under the heads of arithmetical and algebraical intellects. All economical and practical wisdom is an extension or variation of the following arithmetical formula: $2 + 2 = 4$. Every philosophical proposition has the more general character of the expression $a + b = c$. We are mere operatives, empirics, and egotists, until we learn to think in letters instead of figures."

The metric system of measurement is given full and adequate treatment. But it is put by itself in the supplement, to be used or not as school authorities may desire.

Book III is a complete arithmetic, orderly in plan and simple in statement. It is believed that any child of ordinary intelligence can read it alone, and even without a teacher, can obtain from it a fair measure of both the utilitarian and the cultural values of the subject.

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BOOK III

CHAPTER I

THE FUNDAMENTAL PROCESSES

NOTATION AND NUMERATION

1. Write in as many ways as you can the number of the present year.

Read 26,057; MDXCVII.

Writing numbers is called **notation.**

Reading numbers is called **numeration.**

A regular plan of writing or reading numbers is called a **system** of notation or of numeration.

There are two systems of notation and numeration in use in this country: the **Arabic** and the **Roman.**

The Arabic system was invented by the ancient Hindus, the people of India. It was introduced into Europe by the Arabs, hence its name.

The Roman system was used by the ancient Romans.

The Arabic system is the one used in all common business transactions. The Roman system is used chiefly in the numbering of certain chapters and pages of books, and on the faces of clocks and watches.

Read 349; CCCXLIX. Write eighty-eight in both Arabic and Roman notation. Which do you think the better system? Why?

THE ARABIC SYSTEM

2. The characters, or figures, of the Arabic system are 1, 2, 3, 4, 5, 6, 7, 8, 9, called **digits**, and 0, called **zero**, or **cipher**.

Ten ones, or **units** = 1 ten; $10 \times 1 = 10$.
Ten **tens** = 1 hundred; $10 \times 10 = 100$.
Ten **hundreds** = 1 thousand; $10 \times 100 = 1{,}000$.
1,000 **thousands** = 1 million; $1{,}000 \times 1{,}000 = 1{,}000{,}000$.
1,000 **millions** = 1 billion; $1{,}000 \times 1{,}000{,}000 = 1{,}000{,}000{,}000$.

The use of numbers higher than millions is rare, though the following numbers are found by using 1,000 as a multiplier: billions, trillions, quadrillions, quintillions, sextillions, septillions, octillions, nonillions, and decillions. 1 decillion would contain 33 ciphers.

The Arabic system is called a **decimal** system (from the Latin word **decem**, meaning "ten") because it is a system of tens, each place having a value ten times greater or ten times less than the one next to it.

The word *digit* means "finger." It indicates that the decimal system grew out of the custom of counting on the fingers.

Numbers as written by the Arabic system are divided into **periods** of three figures each. According to its position each figure stands for units, tens, or hundreds of its period.

In 3,126,374,201 the value of each figure is as follows:

	MILLIONS			THOUSANDS			UNITS		
Billions	Hundred millions	Ten millions	Millions (units)	Hundred thousands	Ten thousands	Thousands (units)	Hundreds	Tens	Units
3,	1	2	6,	3	7	4,	2	0	1

3. Each period is read as a separate number followed by the name of the period, excepting that the name of the unit's period is not read, but is "understood."

The number above is read: "Three billion, one hundred twenty-six million, three hundred seventy-four thousand, two hundred one."

The cipher is used merely to fill vacant spaces and is not read.

The places of the figures in a number are called **orders.**

In reading numbers from left to right, a figure in unit's place of any period is called a figure of the **first** order. A figure in ten's place is called a figure of the **second** order. A figure in hundred's place is called a figure of the **third** order, and so on.

4. WRITTEN EXERCISES

1. Copy, point off, and read:

1002356.	37027017.
467392.	2080104346.
100247016.	19405007084.
403006040.	700600050200.
830042.	14308012.

2. Express with figures:

7 billion, 80 million, 2 hundred thousand, 50.
20 billion, 3 million, 15 thousand, 8.
215 billion, 0 million, 140 thousand, 4 hundred.
305 million, 1 thousand, 11.
5 million, 5 thousand, 5 hundred, 5.

3. Write in words:

206,937.	60,215,840,075.
92,010,005.	40,003,090.
209,387,016.	1,002,003,040.
587,009,307.	26,807,243,201.

THE ROMAN SYSTEM

5. The Roman notation uses seven letters, namely:

I = 1,	V = 5,	X = 10,	L = 50,
C = 100,	D = 500,	M = 1000.	

These letters are combined to represent other numbers according to the following principles:

(1) When a letter is followed by the same letter or by a letter of less value, it indicates that the values of the letters are added.

Thus, II represents 2; VI, 6; XXX, 30; XV, 15; LX, 60; CX, 110; DC, 600.

(2) When a letter is followed by a letter of greater value, its value is to be taken from that of the greater.

Thus, IV represents 4; IX, 9; XL, 40; XC, 90; CD, 400; CM, 900.

(3) Placing a dash over a letter multiplies its value by 1,000.

Thus, $\overline{V}$ represents 5,000; $\overline{X}$, 10,000; $\overline{IV}$, 4,000.

6. ORAL AND WRITTEN EXERCISES

1. Read the following:

$\overline{M}$.	$\overline{LXXIX}$.	$\overline{CCV}$.	$\overline{MDXC}$.
XCIX.	CXVIII.	DXIX.	MDCCC.
DLXXV.	CCXXVI.	DCXL.	MMDC.
CXXXII.	CDLXIII.	DCCL.	IVCCXL.
DCLIX.	MDCC.	XCVI.	CCCXL.

2. Express the following in the Roman notation:

55.	61.	101.	496.	1,607.
66.	58.	114.	509.	5,000.
79.	99.	325.	1,900.	10,200.
94.	89.	423.	2,821.	16,306.

ADDITION

7. What is the sum of 123 + 456?

What is the process of finding the sum called?

Addition is the process of combining two or more numbers into a single equivalent number, called the **sum.**

The numbers so combined are called **addends.**

1. Add 9 to each of the following: 12, 23, 32, 44, 55, 67.

2. How do you add 9 to numbers larger than 10?

3. Add 8 to each of the following: 13, 24, 35, 46, 57, 66, 83, 87, 94.

4. How do you add 8 to numbers larger than 10?

5. Add 7 to each of the following numbers: 23, 34, 45, 56, 63, 78, 82, 94.

6. How do you add 7 to numbers larger than 10?

7. Add: 6,845
5,385

8. Explain each figure of the sum.

ORAL EXERCISES

8. In the following numbers the dollars are separated from the cents by a line instead of a decimal point. This method is used in books of accounts. Add, while some one times you:

1.	2.	3.	4.
96|48	465|73	875|23	28,675|09
34|73	376|45	3,468|71	86,750|92
65|48	283|91	51|98	67,509|86
84|56	75|60	918|46	75,092|28
45|68	|29	7,326|93	50,928|67
7|15	25|98	3,962|58	92,764|92
5|71	9|17	5,806|73	89,674|85
28|39	724|82	3,785|64	76,658|37
93|82	239|56	97|08	98,747|61

TESTING ADDITION

9. Add and test:

3,184
5,732
6,813
5,940
6,028
27,697

Test the result of the above by the following rule:

Add the columns first upward, then downward. If the two results agree, the sum is *probably* correct.

NOTE. For testing addition 'by casting out the nines,' see page 262.

ORAL EXERCISES

10. Add the following and test:

1.	2.	3.	4.
169	1,306	321	72
724	271	42	777
382	82	1,675	8,888
576	3,574	4,107	9,999
444	16,999	6,010	10,101
555	1,004	7,280	23,326

SUBTRACTION

11. What is the difference of 327 – 226?

Subtraction is the process of finding how many units more or less one number contains than another.

The larger number is called the **minuend**; the smaller is called the **subtrahend.** The excess is called the **difference.**

1. Subtract: 2,601
1,777

2. How do you subtract when a figure in the subtrahend is larger than a figure of the same order in the minuend?

WRITTEN EXERCISES

12. Subtract:

1. 67,023	2. 81,456	3. 103,521	4. 324,510
51,569	29,879	85,762	167,823

5. 367,531 − 12,987.

6. 120,762 − 94,876.

MAKING CHANGE

13. I bought goods amounting to $4.15 and gave a $5 bill.

Count the change by adding aloud; thus, "$4.15, $4.25, $4.50, $5.00."

What coins were used?

Make change for the following, by adding aloud:

	Purchase	Money Given		Purchase	Money Given
1.	$1.25	$ 2.00	6.	$ 0.38	$ 1.00
2.	7.23	10.00	7.	0.26	0.50
3.	8.16	10.00	8.	14.30	15.00
4.	4.10	5.00	9.	40.16	100.00
5.	6.21	10.00	10.	67.20	100.00

TESTING SUBTRACTION

14. To test the correctness of subtraction:

Add the difference to the subtrahend; if the sum equals the minuend, the difference is correct. Explain why.

WRITTEN EXERCISES

15. Subtract the following and test:

1. 67,584	2. $9,230.64	3. 44,445,555
56,784	1,873.79	34,446,666
4. 94,387	5. 3,061.51	6. 68,901,332
29,699	1,982.62	27,072,459

NOTE. For testing subtraction by casting out the nines, see page 262.

FINDING NUMBERS FROM THEIR SUM AND DIFFERENCE

16. One half the sum of the sum and difference of two numbers equals the larger number.

One half the difference of the sum and difference of two numbers equals the smaller number.

$7 + 5 = 12$ (sum).
$7 - 5 = 2$ (difference).
$12 + 2 = 14$ (sum of sum and difference).
$14 \div 2 = 7$ (the larger number).
$12 - 2 = 10$ (difference of sum and difference).
$10 \div 2 = 5$ (the smaller number).

ORAL AND WRITTEN EXERCISES

17. Solve:

1. The sum of two numbers is 80; their difference is 20. What are the numbers?

2. The number of votes cast for two candidates was 3,200. The successful candidate had 200 more than the other. How many votes did each receive?

3. Luther and Richard together have 120 marbles. Luther has 10 marbles more than Richard. How many has each?

4. John and Henry in a relay race ran 7 miles. John ran 2 miles farther than Henry. How far did each run?

5. Mr. Allison gave his sons, Alfred and Charles, $28, giving each $1 for every year of his age. He gave Charles $6 more than Alfred. How old was each boy?

6. Mary and Lucy together picked 66 qt. of strawberries. Mary picked 6 qt. more than Lucy. How many did each pick?

7. John and James have 80 cents. John has 20 cents more than James. How many cents has each?

ADDITION AND SUBTRACTION

WRITTEN EXERCISES

18. The fifteen universities in the United States having the largest number of students in the year 1909 were:

Harvard	5,558	Cornell	5,028
Columbia	6,132	* Wisconsin	4,245
* Chicago	5,487	Yale	3,276
* Northwestern	3,197	Pennsylvania	4,857
* Michigan	5,259	Syracuse	3,284
* California	4,084	* Nebraska	3,402
* Illinois	4,502	New York	3,843
* Minnesota	4,351		

1. How many students were enrolled in Harvard, Yale, and Columbia? How many in Columbia, Cornell, and Syracuse?

2. How many students were enrolled in the eastern universities? How many in the western universities?

(Stars show western universities.)

3. How many were enrolled in the fifteen universities?

4. Which of the western universities had the largest enrollment?

5. How much larger was it than that of each of the other western universities?

6. How many more were enrolled in Columbia than in each of the others?

7. Which had the more students, the eastern or western universities? How many more?

8. January 1, Mr. Brown had in the bank $1263.75. He made the following deposits during the month; $180, $67.73, $180, $24.30, $635.02, $873.60, $765, $937.50.

(1) How much did he deposit?

(2) How much did he have in the bank during the month?

9. February 1, he called for a statement of his account with the bank, and together with the paid checks he received a slip of paper on which were written the amounts of the checks drawn.

(1) Add these amounts to find how much money he had drawn from the bank.

(2) How may the amount which Mr. Brown has in the bank February 1 be found?

(3) Find the amount of his balance in the bank on that date.

$101.50
9.00
88.79
17.00
10.00
25.00
25.00
84.35
3.14
103.15
60.00
10.00
124.20
21.23
2.80
92.75
7.25
212.75
18.93
211.45
1.40
21.00
2.52

10. The country's iron production in tons during three years was as follows:

	1908-9	1907-8	1906-7
July	2,101,579	1,218,129	2,259,682
June	1,929,884	1,088,634	2,231,675
May	1,883,330	1,149,688	2,294,005
April	1,738,877	1,149,602	2,216,558
March	1,836,194	1,228,204	2,226,457
February	1,707,340	1,079,721	2,045,068
January	1,797,560	1,045,250	2,205,607
December	1,740,912	1,234,279	2,236,153
November	1,577,854	1,828,125	2,187,665
October	1,567,198	2,336,972	2,196,808
September	1,418,998	2,163,437	1,970,962
August	1,359,831	2,250,419	1,926,736

(1) What was the total production of iron for each year?

(2) How much larger was the production of 1906–7 than that of each of the other years?

(3) How much greater was the production of 1908–9 than that of 1907–8?

MULTIPLICATION

19. How many days are there in 4 years?

$$365 + 365 + 365 + 365 = 365 \times 4 = 1{,}460.$$

Multiplication is a short method of finding the sum of equal numbers.

Define **multiplier, multiplicand, product.**

The multiplier and the multiplicand are **factors** of the product.

The multiplicand (one of the equal addends) may be a **concrete** number, as 7 days, 34 years. The multiplier is always an **abstract** number and tells simply how many times the multiplicand is taken, or how many addends are to be added.

For example: To find out how many days there are in four weeks, we take 7 days 4 times, not 4 weeks times.

If the number to be multiplied is smaller than the multiplier, it may be simpler to perform the multiplication as if both were abstract numbers and to multiply the larger by the smaller, adding the name afterwards.

Thus: How many days are 365 times 17 days?

SOLUTION:

$$\begin{array}{r} 365 \\ 17 \\ \hline 2555 \\ 365 \\ \hline 6205 \end{array}$$

There are 6,205 days in 365 times 17 days.

20. Multiply \$8,042.30 by 2,060.

SOLUTION:

$$\begin{array}{r} \$8042.30 \\ 20\,60 \\ \hline 48253\,8 \\ 160846 \\ \hline \$16567138.00 \end{array}$$

1. Explain what you do with each cipher in the multiplicand and in the multiplier.

2. Why is 6, the right-hand figure of the second partial product, placed under the 5 of the first?

TESTING MULTIPLICATION

21. To test a product, divide it by either the multiplier or the multiplicand. If it is correct, the quotient should equal the other term.

NOTE. For testing multiplication by casting out the nines, see page 263.

Multiply and test:

1.	2.	3.
67,845	189,763	54,321
372	4,279	12,345

4.	5.
43,768	157,865
9,999	8,769

SHORT METHODS OF MULTIPLICATION

22. TO MULTIPLY BY 5, 50, 500, OR 5,000.

1. How is an integer multiplied by 10? By 100? By 1,000?

2. What is the ratio of 10 to 5? Of 100 to 50? Of 1,000 to 500?

3. Multiply 87,659 by 5.

$$87{,}659 \times 10 = 876{,}590.$$
$$876{,}590 \div 2 = 438{,}295.$$

4. Multiply 8,432 by 50.

$$8{,}432 \times 100 = 843{,}200.$$
$$843{,}200 \div 2 = 421{,}600.$$

5. Multiply 5,763 by 50. By 500.

6. Make a rule for a short method of multiplying by 5. By 50. By 500.

23. To Multiply by 25 and 125.

1. What is the ratio of 100 to 25? Of 1,000 to 125?

2. Make a rule for a short way of multiplying by 25. By 125.

WRITTEN EXERCISES

24. Solve:

1. Multiply 46,753 by 25. By 125.

2. Multiply 436,854 by 125.

3. How many days in 5 years? In 50 years? In 500 years? In 125 years?

4. At $50 apiece, how much will 125 bicycles cost? At $65? At $70.50?

5. Find the cost of:

(1) 37 pencils, at 5 cts. each.
(2) 45 Arithmetics, at 50 cts. each.
(3) 77 Spelling Books, at 25 cts. each.
(4) 3 doz. golf balls, at 50 cts. each.
(5) 45 tennis rackets, at $1.25 each.
(6) 75 chairs, at $5 each.
(7) 3 doz. brooms at 25 cts. each.
(8) 14 hats at $2.50 each.
(9) 18 pairs of shoes at $3.50 per pair.
(10) 2 doz. pairs of gloves at $1.25 per pair.

25. To Multiply by 9; by 99; by 999.

$9 = 10 - 1$; $99 = 100 - 1$; $999 = 1{,}000 - 1$.

Multiply 45 by 9

$9 \times 45 = 10 \times 45 - 45 = 450 - 45 = 405$.

Make a rule for multiplying by 9.

1. Multiply 37 by 9. By 99. By 999.

2. How many ciphers are annexed in multiplying by 99? By 999?

WRITTEN EXERCISES

26. Find the cost of:

1. 16 pencils, at 9 cts. each.
2. 432 packages of breakfast food, at 9 cts. each.
3. 165 straw hats, at 99 cts. each.
4. 37 boys' suits, at \$9.99. each.
5. 43 two-pound boxes of chocolates, at 99 cts. each.

27. To MULTIPLY BY $12\frac{1}{2}$.

1. Multiply 16 by $12\frac{1}{2}$.
2. What is the ratio of $100 : 12\frac{1}{2}$?
3. Make a rule for multiplying by $12\frac{1}{2}$.

WRITTEN EXERCISES

28. Solve:

1. Multiply each of the following numbers by 25 and by $12\frac{1}{2}$: 32, 64, 80, 20, 35, 66, 26, 37, 38, 42, 56, 50, 120, 124, 240, 160, 260, 168.

2. Mr. Wilmot purchased an acre of land at \$$12\frac{1}{2}$ per square rod. Find the cost.

3. Mr. Roberts bought two wood lots. One contained 85 acres, for which he paid \$25 per acre; the other contained 112 acres, for which he paid \$$12\frac{1}{2}$ per acre. Find the cost of each lot.

29. To MULTIPLY BY $33\frac{1}{3}$ AND BY $16\frac{2}{3}$.

1. Multiply 27 by $33\frac{1}{3}$.
2. What is the ratio of $100 : 33\frac{1}{3}$?
3. Make a rule for multiplying by $33\frac{1}{3}$.
4. Multiply 36 by $16\frac{2}{3}$.
5. What is the ratio of $100 : 16\frac{2}{3}$?
6. Make a rule for multiplying by $16\frac{2}{3}$.

WRITTEN EXERCISES

30. Solve:

1. Multiply by $33\frac{1}{3}$ the number of months in a year. Of days in June. Of hours in a day. Of minutes in an hour. Of gills in a gallon. Of quarts in a bushel. Of ounces in a pound. Of rods in a mile.

2. A merchant employs two clerks. To one he pays $\$33\frac{1}{3}$ per week, and to the other $\$16\frac{2}{3}$ per week. What is the yearly salary of each?

3. Mr. Lindsley has a farm of which $33\frac{1}{3}$ acres are sown with oats, 25 acres planted with corn, and $12\frac{1}{2}$ with potatoes. If the oats yield 32 bushels per acre, the corn 41 bushels, and the potatoes 240 bushels, how many bushels of each will he have?

4. If a man earns $\$33\frac{1}{3}$ a week and spends $\$16\frac{2}{3}$, how many dollars will he save in 36 weeks? In one year, if he works every week?

ACCOUNTS

31. A record of business transactions is called an **account.**

A person who owes a debt is called a **debtor,** and a person to whom the debt is owed is called a **creditor.**

An amount due to another is called a **debit.**

An amount due from another is called a **credit.**

The difference between the sum of the debits and the sum of the credits in an account is called the **balance.**

Entering the balance on the smaller side and ruling the account is called closing or **balancing** the account.

In business the closing of an account is usually done at stated times, generally at the end of the month.

When the account is closed for the stated time, the balance is carried over to the following account of the next stated time.

Some abbreviations in common use are:

Debit or debtor . . .	Dr.	Account	Acct.
Credit or creditor . .	Cr.	Merchandise	Mdse.

The following form illustrates a closed account of S. T. Delmar, a farmer, with his grocer, R. L. Falmouth. The value of farm products that Mr. Delmar sells Mr. Falmouth and any cash that he pays are put in the debtor column. They show what Mr. Falmouth owes Mr. Delmar.

The cost of the groceries that Mr. Delmar buys of Mr. Falmouth is put in the creditor column. This column shows what is due Mr. Falmouth from Mr. Delmar.

1906.		R. L. FALMOUTH		*Dr.*	*Cr.*
July	1	By 7 lb. Sugar			48
"	"	" 1 bbl. Flour			5 50
"	5	To 3 bbl. Potatoes	$3.80	11 40	
"	"	By 5 lb. Prunes	12¢		60
"	"	" 2 lb. Crackers	10¢		20
"	8	To 8 doz. Eggs	42¢	3 36	
"	"	"12 lb. Butter	35¢	4 20	
"	12	By 4 lb. Rice	9¢		36
"	"	" 2 doz. Oranges	40¢		80
"	16	To 3 bbl. Apples	$3.85	11 55	
"	"	By 1 bbl. Sugar			18 75
"	20	To 8 bbl. Kindling Wood	18¢	1 44	
"	"	By 6 lb. Lard	15¢		90
"	23	" $2\frac{1}{2}$ lb. Cheese	16¢		40
"	"	" 3 lb. Coffee	35¢		1 05
"	30	" 2 pkg. Cereals	15¢		30
"	31	" 3 cans Peaches	25¢		75
		Balance			1 86
				31 95	31 95
		Balance Carried Forward,		1 86	

What is the sum of the debits in the account?

What is the sum of the credits?

How much is the balance that Mr. Falmouth owes Mr. Delmar?

On which side of the account is it entered? Why?

With what word are the debit items introduced? The credit items?

The same account would appear in R. L. Falmouth's book as follows:

1906.		S. T. DELMAR		*Dr.*	*Cr.*
July	1	To 7 lb. Sugar		48	
"	"	" 1 bbl. Flour		5 50	
"	5	By 3 bbl. Potatoes	$3.80		11 40
"	"	To 5 lb. Prunes	12¢	60	
"	"	" 2 lb. Crackers	10¢	20	
"	8	By 8 doz. Eggs	42¢		3 36
"	"	"12 lb. Butter	35¢		4 20
"	12	To 4 lb. Rice	9¢	36	
"	"	" 2 doz. Oranges	40¢	80	
"	16	By 3 bbl. Apples	$3.85		11 55
"	"	To 1 bbl. Sugar		18 75	
"	20	By 8 bbl. Wood	18¢		1 44
"	"	To 6 lb. Lard	15¢	90	
"	23	" 2½ lb. Cheese	16¢	40	
"	"	" 3 lb. Coffee	35¢	1 05	
"	30	" 2 pkg. Cereals	15¢	30	
"	31	" 3 cans Peaches	25¢	75	
		Balance		1 86	
				31 95	31 95
		Balance			1 86

What do the items in the debtor column show? Those in the creditor column?

WRITTEN EXERCISES

32. Write the accounts of the different persons mentioned in the following transactions, as they would appear in the books:

1. B. H. Graham, a farmer, bought of Joralemon & Co., grocers:

Feb.	6, 1906.	3 gallons molasses	@	$ .75
"	10, "	4 bags flour	"	1.25
"	15, "	2 cans salmon	"	.18
"	18, "	6 pounds butter	"	.35
"	21, "	3 pounds cheese	"	.16
"	23, "	10 pounds sugar	"	.06
"	27, "	2 dozen oranges	"	.40

Sold to Joralemon & Co.:

Feb.	8, 1906.	2 cords wood	@	$5.25
"	18, "	40 pounds chickens	"	.20
"	23, "	10 dozen eggs	"	.32
"	27, "	5 gallons maple syrup	"	.95
"	28, "	12 bushels potatoes	"	.85

2. R. L. Martin sold Charles Miller:

July	1, 1906.	10 pounds rice	@	$ .10
"	5, "	4 dozen bananas	"	.15
"	12, "	2 boxes soap	"	2.75
"	15, "	1 pound pepper	"	.15
"	21, "	2 pounds raisins	"	.16
"	30, "	3 cans peaches	"	.25

Bought of Charles Miller:

July	3, 1906.	5 bushels potatoes	@	$ .95
"	10, "	3 crates strawberries (96 quarts)	"	.08
"	20, "	12 pounds butter	"	.25
"	28, "	5 bushels onions	"	.80
"	28, "	15 dozen eggs	"	.25

3. Kearny & Clark sold F. H. Baldwin:

Sept.	2,	1906.	2 gross matches	@	$3.75
"	9,	"	28 pounds sugar	"	.06
"	14,	"	2 pounds coffee	"	.28
"	20,	"	10 pounds crackers	"	.16
"	23,	"	5 pounds tea	"	.60
"	28,	"	3 dozen oranges	"	.35
"	28,	"	2 dozen lemons	"	.20
"	30,	"	2 cans spice	"	.10
"	30,	"	6 boxes cereal	"	.15

Bought of F. H. Baldwin:

Sept.	10,	1906.	2 barrels apples	@	$2.00	
"	18,	"	16 dozen ears corn	"	.18	
"	21,	"	2 bags oats (5 bushels)	"	.40	per bu
"	27,	"	1 barrel vinegar (40 gallons)	"	.28	per gal.
"	28,	"	20 pounds butter	"	.28	
"	28,	"	15 dozen eggs	"	.24	
"	30,	"	8 bushels potatoes	"	.90	

DIVISION

33. Into how many scores of years can you divide a century?

How many times can you take 20 from 100?

Division is the opposite or *inverse* of multiplication. Division is a short method of finding how many times a given subtrahend may be taken from a number; or how many times a number contains a given number as a factor.

Define *divisor*, *dividend*, *quotient*, *remainder*.

The divisor and the quotient are factors of the dividend.

Express in three ways *6 divided by 2.*

Divide 7,101 by 263.

Explain each step and name each trial divisor and trial dividend.

Is the division exact? How do you write the remainder?

34. Divide \$10,000 by 66.

$$
\begin{array}{r}
\$151.51+ \\
66\overline{)\$10000} \\
66 \\
\hline
340 \\
330 \\
\hline
100 \\
66 \\
\hline
340 \\
330 \\
\hline
100 \\
66 \\
\hline
\end{array}
$$

In dividing cents, if the division is not exact, it is customary to omit the fractional remainder and place the sign + after the cents in the quotient.

TESTING DIVISION

35. To test the accuracy of division, multiply the quotient by the divisor, and to the product add the remainder, if any. What must the result equal? Why?

NOTE. For testing division by casting out the nines, see p. 263.

WRITTEN EXERCISES

36. Solve:

1. Divide and test:

(1) $46\overline{)6175}$ (3) $321\overline{)19874}$ (5) $414\overline{)76853}$

(2) $195\overline{)5273}$ (4) $6789\overline{)66778899}$ (6) $1234\overline{)56789}$

2. The highest mountain on the earth is Mt. Everest in India, 29,002 ft. above sea level. The greatest ocean depth known is 31,614 ft. in the Pacific near Guam. State these measurements in miles.

3. The average daily attendance in all the public schools in New York City in 1909 was 656,619 pupils. The appropriation for schools (exclusive of land and buildings) was $27,470,736.80. How much was the cost per pupil?

4. The average daily attendance in the day schools was 545,218. If the pupils were equally divided among 15,695 teachers, how many pupils did each teacher have?

5. On a test run of 2 automobiles from New York to Boston, a distance of 250 mi., one car consumed 24¾ gal. of gasoline, and the other 33¾ gal. of kerosene. If the gasoline cost 15 cts. per gallon, and the kerosene cost 20 cts. per gallon, what was the cost of each kind of fuel per mile?

6. Following are the amounts of money of different kinds that were in circulation in the United States on August 1, 1909:

Gold coin,	$596,806,435	Subsidiary silver,	$132,857,008
Gold certificates,	806,284,359	Treasury notes of 1890,	4,156,121
Silver dollars,	71,887,688	U. S. notes,	338,928,434
Silver certificates,	477,213,767	National bank notes,	667,947,187

(1) Estimating the population at 80,000,000, what was the amount of each kind of money *per capita* (per head, or per person)?

(2) What was the amount of gold and silver coins to each person?

(3) What was the amount of all kinds of paper money per capita? How much more paper money per capita than coin was there?

(4) If all the money were divided equally among the inhabitants of the United States, how much would each person have?

(5) The yield of tobacco in Virginia in a recent year was 73,000,000 lbs. This was grown on 110,000 acres of land. Find the average number of pounds produced on each acre.

FACTORS AND MULTIPLES

DIVISIBILITY

37. A number is **divisible** by any number that will divide it without a remainder.

An **even** number is one divisible by 2.

An **odd** number is one not divisible by 2.

A number is divisible by 2, if its last figure is an even number or 0.

1. Which of the following numbers are divisible by 2:
246, 347, 890, 2,743, 27,692, 98,310, 93,762?

A number is divisible by 3, if the sum of its digits is divisible by 3.

2. Which of the following numbers are divisible by 3:
250, 251, 365, 368, 1,011, 258, 41,681?

A number is divisible by 4, if the last two digits together are divisible by 4.

3. Which of the following numbers are divisible by 4:
3,133, 3,135, 3,124, 6,782, 2,528, 9,192, 6,788?

A number is divisible by 5, if its last figure is 0 or 5.

4. Write 6 numbers of 3 or more figures that are divisible by 5.

A number is divisible by 6, if it is divisible by the factors of 6 (3 and 2).

5. Which of the following numbers are divisible by 6:
67,830, 9,183, 108, 9,768, 5,454, 123,456, 5988, 76,512?

A number is divisible by 8, if the number made by its last three figures is divisible by 8.

6. Which of the following numbers are divisible by 8:
12,504, 769,432, 467,216, 141,414, 151,528?

A number is divisible by 9, if the sum of its digits is divisible by 9.

7. Which of the following numbers are divisible by 9:
333, 4,788, 67,230, 578,645, 171,819, 654,321, 450,000?

A number is divisible by 10, if its last figure is 0.

8. By what other numbers is a number having 0 for its last figure divisible?

A number is divisible by 12, if it is divisible by 3 and 4.

9. Which of the following numbers are divisible by 12:
16,848, 378,492, 38,495, 167,859, 314,152?

10. By which of the numbers from 2 to 12 is each of the following divisible:
128, 133, 500, 62, 85, 324, 252, 144, 5,278, 365, 1492, 1,908?

11. In how many different ways can a gardener set in equal rows 240 rose bushes? 2,013 asters? 124 dahlias? 1,500 pansies?

A **prime number** is a whole number that is divisible only by itself and 1.

12. Name the prime numbers from 1 to 100.

All whole numbers not prime are **composite.**

Any **exact divisor** of a number is a **factor** of the number.

A **prime factor** is a factor that is a prime number.

13. Give all the factors and the prime factors of each composite number from 1 to 20. From 90 to 100.

14. What is a multiple?

15. What are the factors of 6?

16. What are the products of 6 by 2, 3, 4, 5, 6, 7, 8, 9, and 10?

17. Are 2 and 3 factors of each of these products? Why?

A number that will divide another number without a remainder will divide any product of that number; for example, 6 will divide 12; it will also divide 24, 36, 60, 72.

ORAL EXERCISES

38. Name:

1. 5 products of 8 that contain 4 without a remainder.
2. 5 " " 10 " " 5 " " "
3. 5 " " 14 " " 7 " " "
4. 5 " " 16 " " 8 " " "
5. 5 " " 18 " " 9 " " "

WRITTEN EXERCISES

39. Solve:

1. A certain garden contains 360 sq. ft. Its length is 20 ft. What is its breadth?

SUGGESTION. 360 is a multiple of two factors, of which 20 is one. What is the other?

2. There are 900 sq. ft. in a lawn 30 ft. long. How wide is the lawn?

3. A floor 32 ft. long contains 576 sq. ft. How wide is it?

4. One side of a barn is 23 ft. high from the foundation to the roof, and contains 1035 sq. ft. How long is it?

GREATEST COMMON DIVISOR

40. A **common factor** or **common divisor** of two or more numbers is one that will divide each of them without a remainder.

What is a greatest common divisor (g. c. d.)? Find by inspection the g. c. d. of:

1. 15, 25, 50.
2. 14, 21, 28.
3. 16, 32, 42.
4. 36, 96, 180.
5. 12, 18, 36, 60.
6. 42, 84, 96.
7. 24, 96, 120.
8. 30, 60, 90, 120.

To find the greatest common divisor of numbers not readily factored at sight, the following method may be used.

Find the greatest common divisor of 18, 36, 27, and 45.

SOLUTION:

$$\begin{array}{r} 3)\,18\ \ 36\ \ 27\ \ 45 \\ 3)\,\ 6\ \ 12\ \ \ 9\ \ 15 \\ \hline \ \ \ 2\ \ \ 4\ \ \ 3\ \ \ 5 \end{array}$$

$3 \times 3 = 9$, the greatest common divisor.

The numbers are arranged in a horizontal line and divided by any number, 3, that will divide each of them without a remainder. The quotients are written below and divided by another number, 3, that will divide each of them without a remainder. This is repeated until no number will so divide all the quotients. The product of the divisors, $3 \times 3 = 9$, is the greatest common divisor. Explain.

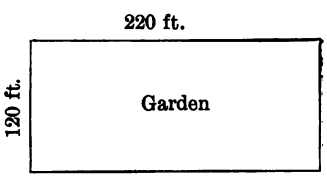
WRITTEN EXERCISES

41. Solve:

1. Find the g. c. d. of:

(1) 50, 175, 125, 400.
(2) 72, 48, 96, 24.
(3) 70, 490, 476.
(4) 180, 195, 990.
(5) 21, 210, 315.
(6) 16, 48, 96, 32.
(7) 144, 1728, 84.
(8) 80, 40, 60, 120.

2. The posts of the fence around the four sides of a garden are set equal distances apart. What is the greatest number of feet possible between the posts (g. c. d.)? Test your answer by a diagram drawn to scale.



3. Three men purchased sheep at the same price per head. One spent \$81; another, \$36; and the third, \$72. What was the greatest price per head that they could pay (g. c. d.)? At that price, how many sheep could all buy?

LEAST COMMON MULTIPLE

42. A number that is divisible by two or more numbers is called a **common multiple** of those numbers. For example, 16 is a common multiple of 2, 4, and 8; 20 is a common multiple of 2, 4, 5, and 10.

1. What is a **least common multiple** (l. c. m.)?

2. Find the l. c. m. of 24, 48, 36.

SOLUTION: The prime factors of 24 are 2, 2, 2, 3.
" " " " 48 " 2, 2, 2, 2, 3.
" " " " 36 " 2, 2, 3, 3.

2 occurs four times in 48. Hence, 2 must be taken four times in finding the l. c. m.

3 occurs twice in 36. Hence, 3 must be taken twice in finding the l. c. m.

$$2 \times 2 \times 2 \times 2 \times 3 \times 3 = 144 = \text{the l. c. m.}$$

The product of the prime factors is the l. c. m. Explain.

To find the least common multiple of two or more numbers, multiply together all their prime factors, taking each the greatest number of times that it occurs in any one of the numbers.

ORAL OR WRITTEN EXERCISES

43. Find by inspection the l. c. m. of:

1. 12, 72, 84.
2. 42, 98, 120.
3. 16, 64, 96.
4. 28, 56, 112.
5. 4, 8, 12, 16.
6. 14, 20, 28.
7. 24, 48, 72.
8. 18, 45, 90.
9. 5, 15, 20.
10. 6, 18, 4.
11. 13, 39, 78.
12. 35, 14, 140.

44. To find the least common multiple of numbers that cannot be readily factored at sight, the following plan may be used:

Find the least common multiple of 6, 9, 12, 15, and 18.

SOLUTION:

2)	6	9	12	15	18
3)	3	9	6	15	9
3)	1	3	2	5	3
	1	1	2	5	1

$2 \times 3 \times 3 \times 2 \times 5 = 180$, the least common multiple.

The numbers are arranged in a horizontal line and divided by the least number, 2, that will divide two or more of them without remainders. The quotients and undivided numbers are written below and divided by the least number, 3, that will divide two or more of them without remainders. This operation is continued until there is no number greater than 1 that will divide two or more of the quotients and undivided numbers without remainders.

The prime factors are the divisors and undivided numbers. Their product is the least common multiple.

When all the numbers are prime numbers, their product is their least common multiple.

WRITTEN EXERCISES

45. Factor and find the l. c. m. of:

1. 4, 6, 8, 12.
2. 5, 8, 10, 16.
3. 6, 9, 27, 18.
4. 2, 3, 4, 8, 6.
5. 12, 15, 8, 10, 30.
6. 20, 45, 30, 15, 60.

46. Find the g. c. d. and the l. c. m. of:

1. 6, 12, 24, 36.
2. 21, 35, 42, 14.
3. 16, 40, 24, 56.
4. 9, 15, 18, 27.
5. 12, 24, 16, 32, 8.
6. 18, 36, 12, 60, 20.
7. 30, 25, 10, 40, 80.
8. 12, 16, 32, 64, 8.

REVIEW

47. Questions on subject-matter.

1. Why is the Arabic system called a decimal system?

2. In what table of denominate numbers is the decimal system used? Would it be better if the decimal system were used in all tables?

3. What is the name of the answer in each of the four different arithmetical processes?

4. Name and give examples of two different classes of numbers.

5. Work a problem in addition, beginning with the left-hand column.

6. Name and make all of the signs used thus far in arithmetic.

7. How would you test the result in a problem in subtraction?

8. How would you test the result in a problem in division?

9. What is a simple number? A compound number?

10. What is an abstract number? A concrete number?

11. In multiplication, which term is always abstract?

12. Why will 5 divide any number that 10 will divide?

13. Why is it that 10 will not divide all numbers that 5 will divide?

14. What is a factor? A prime factor?

15. In what class of problems is a knowledge of factoring important?

16. How many multiples may 2 and 3 have?

17. Can you give a reason for using the least common multiple?

WRITTEN EXERCISES

48. Solve:

1. Express in words the following numbers:
3,001,003, 11,111,101, 4,324,101, 2,002,002.

2. Express in figures the following numbers:
One million, forty thousand, ten.
Twenty-seven million, twenty-seven thousand, twenty-seven.

3. Express in Roman notation the number 5555.

4. Add once only the following numbers and test the result: 30,647, 26,004, 10,118, 96, 386, 92,896, 4379.

5. Multiply once only the following numbers and test the result: 8,604,975 by 64,893.

6. Multiply in two different ways the following numbers: 4680 by 50; 640 by 25.

7. Divide the following numbers by the ordinary method of division, and also by dividing by the factors of the divisor: 8736 by 24.

8. Multiply in two ways the following numbers: 249 by $33\frac{1}{3}$.

9. Make a bill of the following transactions, in which you show the debit and credit items and how the account stands:

March 1, Mr. Gray bought of Mr. Smith 10 lb. of sugar at \$.06 a pound and $\frac{1}{2}$ lb. of tea at \$.90 a pound. He sold Mr. Smith 10 bu. of potatoes at \$.60 a bushel and 2 doz. eggs at \$.30 a dozen.

March 10, Mr. Gray bought of Mr. Smith 5 lb. of coffee at \$.40 a pound, 9 cans of sweet corn at \$.11 a can. He sold him 20 chickens at \$.45 each.

March 20, Mr. Gray bought of Mr. Smith 25 lb. of sugar at \$.05 a pound, 2 lb. of raisins at \$.12 a pound, $1\frac{1}{2}$ lb. of pepper at \$.30 a pound. He sold Mr. Smith 4 doz. eggs at \$.25 a dozen.

10. Copy the following names of various things, and place after each the price as nearly as you can:

Apples, oranges, eggs, tea, sugar, steak, ice, horse, sheep, pair of your shoes, handkerchief, this book.

11. What numbers will divide the following number: 1728? Give a reason for each one named.

12. Find by inspection three multiples of the following numbers: 2, 3, 4, 6, 12.

13. Find by inspection two common divisors of the following numbers: 40, 64, 16, 56, 32.

14. A circular track is 3 miles in circumference. Three men start from the same point to travel around it until they all come together again at the place of starting. A travels 2 miles an hour, B 3 miles an hour, and C 6 miles an hour. How many hours did they travel? How many miles does each travel?

15. A farmer bought three farms which contained 360 acres, 450 acres, and 240 acres. He gave these to his two sons and two daughters, giving to each son twice as much as to each daughter. How many acres did each receive?

16. A man had $6000. He gave to James $\frac{1}{2}$ of this, to Peter $\frac{1}{2}$ of the remainder, to Susan $\frac{1}{2}$ of this remainder, and the rest to Clara. How many dollars did each receive?

SUMMARY OF CHAPTER I

THE FUNDAMENTAL PROCESSES:

Addition, subtraction, multiplication, division, with tests and applications.

FACTORS AND MULTIPLES:

Greatest common divisor and least common multiple.

CHAPTER II

COMMON AND DECIMAL FRACTIONS

FRACTIONS DEFINED

49. Any quantity considered as a single thing, or used as a measure of other quantities, is called a **unit.**

1, 5, 20, a pound, an acre, a foot, are units.

50. A **fraction** is one, or more than one, of the equal parts of a unit, as $\frac{1}{2}$, $\frac{2}{3}$ of a pound, $\frac{3}{4}$ of an acre.

Write a fraction.

Name its numerator and its denominator.

The numerator and the denominator of a fraction are called the **terms** of the fraction.

A fraction whose value is less than 1 is called a **proper fraction,** as $\frac{1}{2}$.

A fraction whose value is 1 or greater than 1 is called an **improper fraction,** as $\frac{6}{3}$.

A number made up of an integer and a fraction written together as one number is called a **mixed number,** as $1\frac{1}{2}$.

51. There are four facts about all fractions that it is well to remember.

(1) Every fraction represents one or more of the equal parts of some unit.

(2) The unit of which the fraction is a part may be any quantity used as a single thing, and not merely the number 1.

For example, $\frac{5}{6}$ may be a fraction of 1 mile or of 30 yards or of an abstract number, as 1 or 24. If the unit is not mentioned, it is commonly understood to be 1.

(3) A fraction may itself be treated as a unit. So we may add, subtract, multiply, and divide fractions as we do whole numbers.

When we say $\frac{1}{2}$ of $\frac{1}{4}$, $\frac{1}{4}$ is treated as a unit.

(4) Every fraction shows a ratio. Its value, considered as a unit, is the quotient of its numerator divided by its denominator.

The value of $\frac{8}{4}$ as a unit is the quotient 2.

REDUCTION

52. REDUCING TO HIGHER OR LOWER TERMS.

Multiply both terms of $\frac{2}{3}$ by 2.

Divide both terms of $\frac{4}{8}$ by 2.

Is the value of the fraction changed in either case? Why?

Multiplying or dividing both terms of a fraction by the same number does not change its value.

Changing the form or the terms of a fraction without changing its value is called **reduction.**

Reduce to their lowest terms:

$$\frac{4}{16},\ \frac{15}{20},\ \frac{125}{1000},\ \frac{1600}{3200},\ \frac{8}{33},\ \frac{111}{444}.$$

Give a rule for this process.

WRITTEN EXERCISES

53. Reduce to lowest terms:

1. $\frac{125}{800},\ \frac{98}{343},\ \frac{81}{162}.$
2. $\frac{56}{128},\ \frac{77}{132},\ \frac{105}{165}.$
3. $\frac{216}{288},\ \frac{567}{891},\ \frac{144}{1728}.$
4. $\frac{375}{1000},\ \frac{256}{768},\ \frac{162}{405}.$

54. REDUCING TO A COMMON DENOMINATOR.

How do you reduce $\frac{1}{3}$ to 12ths?

By what factor must you multiply the terms of $\frac{2}{3}$ to reduce the fraction to 12ths?

Reduce $\frac{3}{4}$ to 8ths; $\frac{9}{10}$ to 20ths.

Give a rule for this process.

Reduce to fractions having a common denominator:

$\frac{1}{3}$, $\frac{2}{9}$, $\frac{3}{4}$, and $\frac{6}{12}$.

Give a rule for this process.

WRITTEN EXERCISES

55. Find a common denominator of:

1. $\frac{1}{5}$, $\frac{2}{3}$, $\frac{5}{9}$, $\frac{2}{15}$.
2. $\frac{2}{7}$, $\frac{3}{4}$, $\frac{1}{2}$, $\frac{4}{5}$.
3. $\frac{3}{10}$, $\frac{2}{9}$, $\frac{5}{6}$, $\frac{1}{5}$.
4. $\frac{1}{7}$, $\frac{2}{3}$, $\frac{1}{6}$, $\frac{5}{21}$.
5. $\frac{5}{6}$, $\frac{2}{5}$, $\frac{1}{10}$, $\frac{7}{15}$.
6. $\frac{2}{3}$, $\frac{5}{9}$, $\frac{2}{6}$, $\frac{4}{27}$.

To reduce fractions having unlike denominators to equal fractions with like denominators, find a common multiple of the denominators and change each fraction to an equal fraction having this common multiple for a denominator.

56. REDUCING TO A LEAST COMMON DENOMINATOR.

How do you find the least common multiple of several numbers?

Find the l. c. m. of 9, 5, 15, 3; 7, 3, 2, 9.

The least common multiple of several denominators is the **least common denominator.**

Find the least common denominator of:

$\frac{3}{160}$, $\frac{1}{80}$, $\frac{5}{320}$, $\frac{3}{40}$; $\frac{2}{9}$, $\frac{4}{5}$, $\frac{7}{15}$; $\frac{5}{7}$, $\frac{2}{3}$, $\frac{8}{21}$, $\frac{7}{9}$.

Give a rule for this process.

WRITTEN EXERCISES

57. Reduce to fractions having the least common denominator:

1. $\frac{12}{13}$, $\frac{4}{39}$, $\frac{7}{65}$, $\frac{8}{78}$, $\frac{3}{52}$.
2. $\frac{4}{15}$, $\frac{3}{25}$, $\frac{17}{40}$, $\frac{13}{50}$, $\frac{6}{35}$.
3. $\frac{5}{24}$, $\frac{3}{8}$, $\frac{16}{32}$, $\frac{4}{12}$, $\frac{7}{60}$.
4. $\frac{12}{1000}$, $\frac{4}{5000}$, $\frac{7}{2000}$, $\frac{9}{1500}$.
5. $\frac{3}{20}$, $\frac{1}{30}$, $\frac{4}{15}$, $\frac{7}{60}$, $\frac{9}{40}$.
6. $\frac{6}{15}$, $\frac{8}{25}$, $\frac{10}{75}$, $\frac{3}{5}$, $\frac{2}{3}$.

58. REDUCING IMPROPER FRACTIONS TO MIXED NUMBERS.

Reduce $\frac{11}{8}$ to a mixed number. Give a rule for this process.

ORAL EXERCISES

59. Reduce to whole or mixed numbers:

1. $\frac{15}{2}$, $\frac{25}{3}$, $\frac{36}{5}$, $\frac{41}{7}$, $\frac{47}{9}$.
2. $\frac{51}{6}$, $\frac{59}{9}$, $\frac{62}{12}$, $\frac{65}{13}$, $\frac{71}{11}$.
3. $\frac{120}{4}$, $\frac{115}{3}$, $\frac{201}{2}$, $\frac{213}{3}$, $\frac{204}{7}$.

WRITTEN EXERCISES

60. Reduce to whole or mixed numbers:

1. $\frac{61}{12}$, $\frac{7}{2}$, $\frac{125}{40}$, $\frac{875}{37}$, $\frac{14}{3}$, $\frac{185}{45}$.
2. $\frac{92}{30}$, $\frac{764}{164}$, $\frac{15}{14}$, $\frac{96}{12}$, $\frac{1472}{1400}$.
3. $\frac{176}{9}$, $\frac{185}{12}$, $\frac{217}{8}$, $\frac{291}{5}$, $\frac{372}{9}$.

61. REDUCING MIXED NUMBERS TO IMPROPER FRACTIONS.

Reduce $3\frac{1}{2}$ to an improper fraction.

Give a rule for this process.

ORAL EXERCISES

62. Reduce to improper fractions:

1. $4\frac{1}{3}$, $5\frac{1}{6}$, $6\frac{2}{3}$, $8\frac{3}{5}$, $12\frac{3}{7}$, $11\frac{3}{8}$.
2. $6\frac{2}{5}$, $7\frac{5}{9}$, $10\frac{3}{7}$, $9\frac{5}{12}$, $13\frac{2}{3}$, $15\frac{1}{5}$.

WRITTEN EXERCISES

63. Reduce to improper fractions:

1. $16\frac{1}{2}$, $7\frac{2}{3}$, $85\frac{1}{15}$, $14\frac{1}{14}$, $7\frac{1}{7}$, $18\frac{1}{3}$.
2. $125\frac{1}{4}$, $16\frac{1}{4}$, $73\frac{1}{2}$, $88\frac{1}{8}$, $91\frac{3}{5}$, $195\frac{1}{4}$.
3. $402\frac{2}{3}$, $425\frac{4}{5}$, $501\frac{1}{7}$, $475\frac{5}{6}$, $650\frac{3}{8}$.

ADDITION

64. Add $\frac{1}{5}+\frac{2}{5}+\frac{3}{5}+\frac{4}{5}$.

How do you add fractions having *like* denominators?

Add $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{5}{12}$.

In adding fractions having unlike denominators, it is sometimes easier to take the product of the denominators of the fractions for a common denominator than to find their least common denominator.

Example: $\frac{2}{3}+\frac{4}{5}+\frac{3}{4}+\frac{1}{2}=\frac{80+96+90+60}{120}=\frac{326}{120}$.

$=2\frac{86}{120}=2\frac{43}{60}$.

The product of the denominators equals 120. This is a common denominator, but not the least common denominator.

The first numerator is $2\times5\times4\times2=80$

" second " " $4\times3\times4\times2=96$

" third " " $3\times3\times5\times2=90$

" fourth " " $1\times3\times5\times4=60$

To add fractions having unlike denominators, reduce to like fractions having a common denominator; add the numerators and place the sum over the denominator.

ORAL EXERCISES

65. Add, finding either the least common denominator or any other common denominator:

1. $\frac{4}{5}+\frac{3}{15}+\frac{7}{30}+\frac{9}{60}$.
2. $\frac{1}{120}+\frac{3}{20}+\frac{2}{5}+\frac{7}{15}$.
3. $\frac{4}{7}+\frac{1}{35}+\frac{3}{10}$.
4. $\frac{5}{12}+\frac{7}{8}+\frac{9}{24}+\frac{2}{3}+\frac{1}{4}$.
5. $\frac{1}{2}+\frac{3}{5}+\frac{6}{8}+\frac{4}{10}$.
6. $\frac{2}{3}+\frac{3}{9}+\frac{2}{5}+\frac{7}{15}$.
7. $\frac{3}{4}+\frac{6}{7}+\frac{5}{14}+\frac{9}{28}$.

SUBTRACTION

66. Subtract $\frac{2}{9}$ from $\frac{7}{9}$.

How do you subtract fractions having like denominators?

67. Subtract $\frac{3}{4}$ from $\frac{5}{6}$.

When fractions have small denominators it is often easier to multiply these together for a common denominator than to find their l. c. d.

68. Subtract $\frac{3}{8}$ from $\frac{5}{6}$.

SOLUTION: $6 \times 8 = 48$.

$3 \times 6 = 18$. $\frac{40-18}{48} = \frac{22}{48} = \frac{11}{24}$.

$5 \times 8 = 40$.

If fractions have unlike denominators, reduce them to fractions having a common denominator before subtracting the numerators.

ORAL EXERCISES

69. Subtract:

1. $\frac{2}{3} - \frac{1}{4}$.	6. $9 - \frac{3}{4}$.	11. $\frac{5}{8} - \frac{1}{2}$.	16. $11 - \frac{7}{8}$.
2. $\frac{1}{2} - \frac{1}{3}$.	7. $5 - \frac{5}{8}$.	12. $\frac{7}{12} - \frac{1}{3}$.	17. $4 - \frac{5}{12}$.
3. $\frac{1}{3} - \frac{1}{6}$.	8. $10 - \frac{3}{5}$.	13. $\frac{2}{3} - \frac{7}{12}$.	18. $3 - \frac{8}{11}$.
4. $\frac{1}{2} - \frac{1}{4}$.	9. $12 - \frac{2}{3}$.	14. $7 - \frac{8}{11}$.	19. $\frac{18}{25} - \frac{2}{5}$.
5. $6 - \frac{4}{5}$.	10. $\frac{4}{8} - \frac{3}{4}$.	15. $8 - \frac{9}{10}$.	20. $\frac{140}{225} - \frac{1}{3}$.

WRITTEN EXERCISES

70. Solve:

1. $\frac{3}{4} - \frac{1}{2} + \frac{2}{3}$.

SUGGESTION: $\frac{9-6+8}{12}$.

2. $\frac{15}{20} + \frac{2}{5} - \frac{3}{4} - \frac{3}{10}$.
3. $\frac{5}{6} - \frac{3}{24} - \frac{2}{12} + \frac{1}{2} - \frac{1}{3}$.
4. $\frac{5}{17} - \frac{3}{20}$.
5. $\frac{16}{21} - \frac{1}{3} + \frac{2}{25}$.
6. $\frac{5}{12} + \frac{3}{18} - \frac{1}{9} - \frac{1}{3}$.
7. $\frac{9}{16} - \frac{3}{8} + \frac{30}{32} - \frac{3}{4}$.
8. $\frac{4}{17} + \frac{5}{7} - \frac{1}{3}$.
9. $\frac{9}{25} + \frac{6}{35} + \frac{2}{5} + \frac{7}{40}$.
10. $\frac{2}{7} + \frac{3}{14} + \frac{15}{49} - \frac{1}{2}$.

11. Mr. Jones owns $\frac{18}{32}$ of the business known as Jones & Co. Of his partners, Mr. Alfred owns $\frac{1}{4}$, Mr. Swift $\frac{1}{8}$, Mr. Allen $\frac{1}{16}$. How much more does Mr. Jones own than each of his partners?

12. How much more does he own than his three partners together?

13. $\frac{3}{5}$ of the road from Smithville to Jonesboro is good, $\frac{1}{3}$ is fair, and the remainder is bad. What part of the road is bad?

14. How much greater is the fractional part of the road that is good than the part that is fair? Than the part that is bad? Than both together?

15. As the result of a drought last year, Mr. Knapp's crops were less than the average, as follows: hay, $\frac{1}{2}$; oats, $\frac{1}{3}$; apples, $\frac{3}{5}$; potatoes, $\frac{3}{8}$; berries, $\frac{2}{3}$. Which crop suffered the greatest loss?

16. How much larger was the fractional loss on this crop than that on each of the others?

17. A field measures $\frac{7}{9}$ of a mile on one side, $\frac{5}{6}$ of a mile on another side, $\frac{8}{11}$ of a mile on the third side, and $\frac{4}{5}$ of a mile on the fourth side. How far is it around the farm?

18. Mary earned $\$\frac{5}{8}$ on Monday, $\$\frac{9}{11}$ on Tuesday, $\$1\frac{1}{12}$ on Wednesday, and $\$\frac{7}{10}$ on Thursday. How much did she earn in all?

19. John weeded the garden in four hours. The first hour he did $\frac{2}{11}$ of it, the second hour $\frac{3}{10}$, the third hour $\frac{1}{6}$. How much was left to be done the fourth hour?

20. Fred had $\frac{9}{10}$ of a dollar. He spent $\frac{5}{6}$ of it for a necktie. What part of the dollar did he have left?

21. Clara made three bows for her dress. For one she used $\frac{13}{15}$ of a yard of ribbon, for the second $\frac{15}{16}$ of a yard, and for the third $\frac{7}{8}$ of a yard. How many yards did she use in all?

MULTIPLICATION AND DIVISION

71. $\frac{4}{2} = 2$. If we multiply the numerator by 4, we have $\frac{4 \times 4}{2} = 8$. 8 = the ratio, 2, multiplied by 4.

How is the value of the fraction affected by multiplying its numerator by 4?

$\frac{6}{2} = 3$. Divide the numerator by 3. We have $\frac{6 \div 3}{2} = \frac{2}{2} = 1$. 1 = the ratio, 3, divided by 3.

How is the value of the fraction affected by dividing its numerator by 3?

Multiplying or dividing the numerator of a fraction by any number multiplies or divides the value of the fraction of that number. Explain.

Multiply $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, $\frac{9}{10}$, $\frac{10}{11}$, $\frac{11}{12}$ by 2, 3, 4, 5, 6, 7, 8, 9 by multiplying the numerators.

Divide $\frac{4}{5}$, $\frac{6}{7}$, $\frac{8}{15}$, $\frac{10}{11}$, $\frac{12}{17}$, $\frac{20}{21}$, $\frac{100}{101}$ by 2 by dividing the numerators.

72. $\frac{4}{4} = 1$. Divide the denominator by 2.

$\frac{4}{4 \div 2} = \frac{4}{2} = 2$. 2 = the ratio, 1, multiplied by 2. Explain.

How is the value of the fraction affected?

Dividing the denominator of a fraction by a number multiplies the value of the fraction by that number. Explain.

$\frac{4}{2} = 2$. Multiply the denominator by 2.

$\frac{4}{2 \times 2} = \frac{4}{4} = 1$. 1 = the ratio, 2, divided by 2.

How is the value of the fraction affected?

Multiplying the denominator of a fraction by a number divides the value of the fraction by that number. Explain.

Multiply and then divide the denominators of the following fractions, and state the effects:

$\frac{1}{6}$ by 2; $\frac{3}{4}$ by 4; $\frac{9}{12}$ by 6; $\frac{50}{100}$ by 10.

WRITTEN EXERCISES

73. Multiply each of the following fractions,

(1) By multiplying the numerators by the given number,

(2) By dividing the denominators:

1. $\frac{3}{4}, \frac{5}{8}, \frac{9}{10}, \frac{11}{12} \times 2.$
2. $\frac{2}{3}, \frac{5}{6}, \frac{7}{9}, \frac{5}{12}, \frac{11}{15}, \frac{7}{18} \times 3.$
3. $\frac{3}{4}, \frac{5}{8}, \frac{7}{16}, \frac{21}{24}, \frac{9}{36} \times 4.$
4. $\frac{4}{5}, \frac{3}{10}, \frac{7}{20}, \frac{12}{30}, \frac{14}{25} \times 5.$
5. $\frac{5}{6}, \frac{8}{12}, \frac{7}{24}, \frac{9}{36}, \frac{5}{18} \times 6.$
6. $\frac{6}{7}, \frac{8}{14}, \frac{9}{21}, \frac{20}{28}, \frac{15}{35} \times 7.$
7. $\frac{7}{8}, \frac{15}{16}, \frac{18}{24}, \frac{7}{32}, \frac{25}{40} \times 8.$
8. $\frac{8}{9}, \frac{5}{18}, \frac{12}{27}, \frac{19}{36}, \frac{14}{45} \times 9.$

Divide each of the following fractions,

(1) By dividing the numerators,

(2) By multiplying the denominators by the given number:

1. $\frac{4}{5}, \frac{6}{7}, \frac{2}{3}, \frac{8}{7}, \frac{4}{11}, \frac{10}{12} \div 2.$
2. $\frac{3}{4}, \frac{6}{7}, \frac{9}{10}, \frac{6}{11}, \frac{3}{8}, \frac{12}{16}, \div 3.$
3. $\frac{4}{5}, \frac{8}{16}, \frac{12}{13}, \frac{16}{21}, \frac{24}{30}, \frac{16}{38} \div 4.$
4. $\frac{6}{7}, \frac{12}{17}, \frac{18}{21}, \frac{24}{25}, \frac{30}{33}, \frac{12}{22} \div 6.$
5. $\frac{8}{9}, \frac{16}{19}, \frac{32}{41}, \frac{40}{42}, \frac{16}{15}, \frac{16}{20} \div 8.$
6. $\frac{5}{8}, \frac{10}{12}, \frac{5}{7}, \frac{15}{24}, \frac{25}{45}, \frac{10}{21} \div 5.$

MULTIPLICATION

74. Multiply $\frac{3}{4}$ by 2.

$$\frac{3}{4} \times 2 = \frac{3 \times 2}{4} = \frac{6}{4} = 1\frac{1}{2}, \text{ or } \frac{3}{4 \div 2} = \frac{3}{2} = 1\frac{1}{2}.$$

To multiply a fraction by a whole number, multiply the numerator or divide the denominator of the fraction by the whole number.

Multiply $\frac{1}{5}$ by 5; $\frac{2}{6} \times 3$; $4 \times \frac{15}{16}$.

Mr. Avery has 5 gardens containing each $\frac{15}{16}$ of an acre. How many acres in them all?

What is $\frac{1}{2}$ of $\frac{1}{4}$? Show this by a drawing.

In what other way can you obtain the result?

Multiply $\frac{3}{4} \times \frac{2}{4} \times \frac{6}{7}$.

SOLUTION:
$$\frac{3}{4} \times \frac{2}{4} \times \frac{6}{7} = \frac{3}{\cancel{4}_{2}} \times \frac{\cancel{2}^{1}}{\cancel{4}_{2}} \times \frac{\cancel{6}^{3}}{7}$$

$$= \frac{3 \times 3}{2 \times 2 \times 7} = \frac{9}{28}.$$

In the second example we cancel **2**, a factor of 4, in the denominator of $\frac{3}{4}$ and in the numerator of $\frac{2}{4}$. Then we cancel **2**, a factor of 4, in the denominator of $\frac{2}{4}$, and also **2**, a factor of 6, in the numerator of $\frac{6}{7}$.

To multiply a fraction by a fraction, cancel all factors common to both numerators and denominators, and multiply the remaining factors of the numerators together for the new numerator, and those of the denominators for the new denominator.

Since to multiply a fraction by a fraction is to take a fraction of a fraction, it follows that when two proper fractions are multiplied together the value of the product is always less than that of either factor.

$\frac{4}{5} \times \frac{1}{2} = \frac{\overset{2}{\cancel{4}}}{5} \times \frac{1}{\cancel{2}} = \frac{2}{5}$. $\frac{2}{5}$ is less than either $\frac{4}{5}$ or $\frac{1}{2}$.

WRITTEN EXERCISES

75. Multiply:

1. $\frac{3}{8} \times \frac{2}{5} \times \frac{10}{18} \times \frac{4}{5} =$
2. $\frac{1}{2} \times \frac{6}{7} \times \frac{14}{15} \times \frac{5}{8} =$
3. $\frac{1}{5} \times \frac{10}{18} \times \frac{26}{30} =$
4. $\frac{2}{3} \times \frac{9}{10} \times \frac{5}{6} \times \frac{7}{8} \times \frac{16}{28} =$

5. Mr. Thomas divided $\frac{1}{2}$ of a dollar equally among 5 children; what part of a dollar did he give each?

6. Mr. Dalrymple sowed $\frac{1}{2}$ ton of wheat on five fields: he sowed $\frac{1}{6}$ of it on field A; $\frac{1}{3}$ on field B; $\frac{1}{5}$ on field C; $\frac{2}{15}$ on field D; and the remainder on field E. What part of a ton did he sow on each?

7. In Mr. Whitman's berry patch there are $\frac{11}{16}$ of an acre. Of this $\frac{1}{2}$ was in blackberries, $\frac{2}{5}$ in raspberries, and the remainder in strawberries. What part of an acre was used for each kind of berries?

8. Our athletic field is $\frac{5}{20}$ of a mile long and $\frac{4}{15}$ of a mile wide. Its area is what part of a square mile? How many acres does it contain?

DIVISION

76. Divide:

$\frac{4}{5} \div 2 = \frac{2}{5}$; or $\frac{1}{2}$ of $\frac{4}{5} = \frac{4}{10} = \frac{2}{5}$.

To divide a fraction by a whole number, divide the numerator or multiply the denominator by the number. (See p. 39.)

Divide: $\frac{4}{9} \div \frac{2}{9} = 4 \div 2 = 2.$

Test: $2 \times \frac{2}{9} = \frac{4}{9}$.

To divide a fraction by another fraction having the same denominator, divide the numerator of the dividend by the numerator of the divisor.

Divide $\frac{1}{3}$ by $\frac{1}{12}$.

$\frac{1}{3} \div \frac{1}{12} = \frac{4}{12} \div \frac{1}{12} = 4 \div 1 = 4.$

To divide a fraction by a fraction having an unlike denominator, reduce the fractions to a common denominator and divide the numerator of the dividend by that of the divisor.

Divide $\frac{4}{5}$ by $\frac{3}{4}$.

$\frac{4}{5} \div \frac{3}{4} = \frac{16}{20} \div \frac{15}{20} = \frac{16}{15} = 1\frac{1}{15}$.

ORAL EXERCISES

77. Divide:

$\frac{6}{7}$ by 3; $\frac{3}{8}$ by 4; $\frac{5}{6}$ by $\frac{1}{6}$; $\frac{15}{16}$ by $\frac{3}{16}$; $\frac{1}{2}$ by $\frac{1}{5}$; $\frac{99}{100}$ by $\frac{9}{10}$.

A SHORT METHOD OF DIVISION

78. The product of the denominators is always a common denominator, though it may not be the least common denominator. Explain why.

The new numerator in each case is the product of the given numerator by the denominator of the other fraction. Explain.

What will be the new numerator of each of the fractions in the following divisions?

$\frac{2}{3} \div \frac{1}{2}$; $\frac{5}{6} \div \frac{2}{7}$; $\frac{4}{7} \div \frac{3}{5}$; $\frac{6}{7} \div \frac{3}{4}$.

After having found the new numerators, the denominator is of no further use. We merely divide the numerators.

Hence, as we can find the new numerators by multiplying each numerator by the denominator of the other fraction, we may express their division without naming the common denominator.

Thus: $\frac{2}{3} \div \frac{1}{2} = \frac{2\times 2}{3\times 2} \div \frac{1\times 3}{2\times 3}$

$= \frac{2\times 2}{1\times 3} = \frac{4}{3} = 1\frac{1}{3}$.

For convenience, we may invert the divisor and then multiply as in multiplication of fractions.

Thus: $\frac{2}{3} \div \frac{1}{2} = \frac{2}{3} \times \frac{2}{1} = \frac{4}{3} = 1\frac{1}{3}$.

From this we have the following rule:

To divide a fraction by a fraction, invert the divisor and proceed as in multiplication of fractions.

WRITTEN EXERCISES

79. Divide:

1. $\frac{7}{8} \div \frac{3}{4}$.
2. $\frac{1}{3} \div \frac{1}{4}$.
3. $2\frac{1}{2} \div \frac{3}{5}$.
4. $\frac{9}{20} \div \frac{7}{15}$.
5. $\frac{9}{10} \div \frac{3}{20}$.
6. $\frac{1}{4} \div \frac{1}{3}$.
7. $\frac{5}{8} \div 3\frac{1}{3}$.
8. $\frac{7}{10} \div \frac{2}{7}$.
9. $\frac{4}{7} \div \frac{5}{10}$.
10. $\frac{7}{8} \div \frac{9}{10}$.
11. $\frac{4}{9} \div \frac{5}{6}$.
12. $\frac{3}{4} \div \frac{7}{12}$.

80. DIVIDING A WHOLE NUMBER BY A FRACTION OR THE REVERSE.

$6 \div \frac{3}{4} = \frac{24}{4} \div \frac{3}{4} = 8$; or $\frac{6}{1} \div \frac{3}{4} = \frac{6}{1} \times \frac{4}{3} = 8$.

To divide a whole number by a fraction, or a fraction by a whole number, reduce the whole number to a fraction by writing 1 for its denominator and divide.

ORAL EXERCISES

81. Read, supplying the quotients:

1. $5 \div \frac{2}{3} =$
2. $\frac{5}{6} \div 7 =$
3. $14 \div \frac{5}{42} =$
4. $\frac{2}{3} \div 5 =$
5. $9 \div \frac{3}{10} =$
6. $8 \div \frac{3}{4} =$
7. $6 \div 1\frac{8}{24} =$
8. $16 \div \frac{8}{8} =$
9. $\frac{3}{4} \div 12 =$

82. The division of one integer by another can be expressed in various ways; as,

$$(a)\ 3 \div 2, \quad (b)\ 3 : 2, \quad (c)\ \tfrac{3}{2}.$$

The division of one fraction by another may be expressed in the same ways; as,

$$(a)\ \tfrac{1}{2} \div \tfrac{1}{4}, \quad (b)\ \tfrac{1}{2} : \tfrac{1}{4}, \quad (c)\ \frac{\frac{1}{2}}{\frac{1}{4}}.$$

Form (c) is sometimes called a **complex fraction**, which is defined as a fraction in whose numerator or denominator, or both, a fraction occurs.

Complex fractions should be rewritten in the common form for division before dividing.

Thus: $$\frac{\frac{1}{2}}{\frac{1}{4}} = \tfrac{1}{2} \div \tfrac{1}{4} = \tfrac{1}{2} \times \tfrac{4}{1} = \tfrac{4}{2} = 2.$$

$$\frac{2}{\frac{1}{3}} = 2 \div \tfrac{1}{3} = 2 \times \tfrac{3}{1} = 6.$$

The same result will be produced by multiplying the outside terms together for the new numerator and the inside terms for the new denominator.

Thus: $$\frac{\frac{1}{2}}{\frac{1}{4}} = \frac{4 \times 1}{2 \times 1} = \frac{4}{2} = 2.$$

83. Change to the common form and divide the following:

$$\frac{\frac{3}{4}}{\frac{5}{6}};\ \frac{\frac{5}{6}}{\frac{4}{8}};\ \frac{\frac{2}{3}}{\frac{7}{8}};\ \frac{\frac{8}{9}}{\frac{10}{12}};\ \frac{\frac{7}{8}}{\frac{9}{15}}.$$

If mixed numbers occur in either term of a complex fraction, they should be reduced to improper fractions before dividing.

NOTE. The term *complex fraction* has no value to children studying arithmetic; it is an algebraic form. While it may be well for them to understand the meaning of the term, the simple method of treatment here given is to be preferred to the more difficult and confusing ones sometimes used.

WRITTEN EXERCISES

84. Divide:

1. $\frac{1}{15} \div \frac{2}{3}$.
2. $\frac{6}{7} \div \frac{7}{6}$.
3. $\frac{8}{11} \div \frac{2}{3}$.
4. $\frac{4}{7} \div \frac{9}{10}$.
5. $\frac{5}{6} \div \frac{2}{3}$.
6. $\frac{2}{3} \div \frac{5}{12}$.
7. $\frac{18}{21} \div \frac{8}{7}$.
8. $\frac{15}{16} \div \frac{1}{4}$.
9. $\frac{7}{12} \div \frac{5}{16}$.
10. $\frac{21}{25} \div 2\frac{1}{4}$.
11. $\frac{32}{5} \div 6\frac{1}{7}$.

12. $\frac{2}{3}$ of the stock of a railroad is owned by 5 men in equal portions. What part of the stock does each man own?

13. How many boards $\frac{1}{5}$ of a foot wide will be required for a floor 15 ft. wide?

14. How many strips $\frac{1}{15}$ of a foot wide will be required for a mat $\frac{5}{6}$ of a foot wide?

15. Mr. Avery willed $\frac{2}{5}$ of his estate to certain charities, $\frac{1}{20}$ of the estate to each. How many charities profited?

16. A boy rowed $\frac{11}{12}$ of a mile in an hour. How many hours did it take him to row 6 miles?

17. If Frank can do $\frac{3}{10}$ of a piece of work in an hour, how many hours will it take him to do $\frac{11}{18}$ of it?

18. Dorothy is making silk bags from ribbon. For each bag she uses $\frac{5}{9}$ of a yard of ribbon. How many bags can she make from 10 yards?

19. John had $\$\frac{4}{5}$ which he divided among some boys, giving $\$\frac{1}{10}$ to each. To how many boys did he give the money?

20. Mr. Ewing used $\frac{3}{17}$ of a pound of grass seed on each square yard of his lawn. The lawn contained 27 sq. yd. Find the number of pounds that he used on the entire lawn.

21. At a certain school $\frac{7}{8}$ of an acre of land was given to the children for school gardens. $\frac{1}{40}$ of an acre was given to each child. How many children were there?

ADDITION OF MIXED NUMBERS

85. Mixed numbers may be added, either by adding the integers and fractions separately, or by first reducing the mixed numbers to improper fractions.

(1) By adding the integers and the fractions separately.

$2\frac{1}{5} + 4\frac{1}{5} + 5\frac{2}{5}$.

SOLUTION:

$2 + 4 + 5 = 11$

$\frac{1}{5} + \frac{1}{5} + \frac{2}{5} = \frac{4}{5}$,

$11 + \frac{4}{5} = 11\frac{4}{5}$

$$\begin{array}{r} 2\frac{1}{5} \\ 4\frac{1}{5} \\ 5\frac{2}{5} \\ \hline 11\frac{4}{5} \end{array}$$

(2) By first reducing to improper fractions.

$2\frac{1}{4} + 2\frac{1}{3} + 6\frac{1}{2}$.

SOLUTION: $2\frac{1}{4} + 2\frac{1}{3} + 6\frac{1}{2} = \frac{9}{4} + \frac{7}{3} + \frac{13}{2} =$

$\frac{27}{12} + \frac{28}{12} + \frac{78}{12} = \frac{133}{12} = 11\frac{1}{12}$.

WRITTEN EXERCISES

86. Solve:

1. $9\frac{3}{8} + 7\frac{3}{10} + 8\frac{1}{4}$.
2. $10\frac{3}{8} + 12\frac{1}{2} + 11\frac{3}{4}$.
3. $25\frac{1}{8} + 30\frac{3}{4} + 42\frac{5}{16}$.
4. $21\frac{5}{8} + 17\frac{2}{3} + 13\frac{1}{4}$.
5. $16\frac{3}{8} + 14\frac{5}{12} + 35\frac{7}{24}$.
6. $72\frac{5}{9} + 20\frac{2}{3} + 18\frac{1}{6}$.

7. Mr. Leonard sold $125\frac{3}{4}$ pounds of sugar on Monday; $76\frac{5}{8}$ pounds on Tuesday, and $96\frac{9}{16}$ pounds on Wednesday. How many pounds did he sell during the 3 days?

8. Mr. Durand sold $46\frac{11}{12}$ dozen eggs to Mrs. Bates; $18\frac{3}{4}$ dozen to Mrs. Luke, and $35\frac{5}{6}$ dozen to Mrs. Gray. How many dozen eggs did he sell?

9. Mr. Bowers sold 4 plots of land. The first contained $17\frac{1}{2}$ acres; the second contained $25\frac{3}{4}$ acres; the third contained $35\frac{7}{8}$ acres; and the fourth contained $45\frac{7}{16}$ acres. How many acres did he sell in all?

SUBTRACTION OF MIXED NUMBERS

87. To Subtract a Mixed Number from a Whole Number.

From 6 take $4\frac{1}{3}$.

Solution:
$$6 - 4\tfrac{1}{3} = 5\tfrac{3}{3} - \tfrac{1}{3} - 4$$
$$5\tfrac{3}{3} - \tfrac{1}{3} = 5\tfrac{2}{3}$$
$$5\tfrac{2}{3} - 4 = 1\tfrac{2}{3}$$

In subtracting a mixed number from a whole number, we subtract first the fraction, then the whole number.

From the minuend 6, we take 1 unit and change it to $\frac{3}{3}$. We then have $5\frac{3}{3} - 4\frac{1}{3} = 1\frac{2}{3}$.

88. To Subtract a Mixed Number from a Mixed Number.

(1) When the fraction of the minuend is larger than that of the subtrahend.

From $15\frac{5}{6}$ take $9\frac{3}{8}$.

Solution:
$$15\tfrac{5}{6} = 15\tfrac{20}{24}$$
$$9\tfrac{3}{8} = 9\tfrac{9}{24}$$
$$6\tfrac{11}{24}$$

(2) When the fraction of the subtrahend is larger than that of the minuend.

From $12\frac{3}{4}$ take $6\frac{5}{6}$.

Solution:
$$12\tfrac{3}{4} = 12\tfrac{9}{12} = 11\tfrac{21}{12}$$
$$6\tfrac{5}{6} = 6\tfrac{10}{12} = 6\tfrac{10}{12}$$
$$5\tfrac{11}{12}$$

We cannot take $\frac{10}{12}$ from $\frac{9}{12}$, so we "borrow" 1 from 12, the integer of the minuend, and change it to 12ths. $1 = \frac{12}{12}$. $\frac{12}{12} + \frac{9}{12} = \frac{21}{12}$. $\frac{21}{12} - \frac{10}{12} = \frac{11}{12}$. The difference of the integers, $11 - 6 = 5$. The answer is $5\frac{11}{12}$.

To subtract a mixed number from a mixed number reduce the mixed numbers to improper fractions and subtract; or, subtract the fractions and the integers separately.

WRITTEN EXERCISES

89. Solve:

1. $12\frac{3}{4} - 8\frac{5}{8}$.
2. $16\frac{5}{6} - 9\frac{3}{5}$.
3. $20\frac{9}{12} - 12\frac{4}{5}$.
4. $25\frac{9}{16} - 17\frac{1}{4}$.
5. $30\frac{11}{12} - 20\frac{5}{6}$.
6. $42\frac{9}{14} - 25\frac{3}{7}$.
7. $19\frac{1}{2} - 12\frac{1}{4}$.
8. $60\frac{4}{5} - 40\frac{3}{4}$.
9. $75\frac{3}{4} - 54\frac{2}{3}$.
10. $16\frac{13}{16} - 8\frac{3}{8}$.
11. $20\frac{3}{8} - 16\frac{5}{6}$.
12. $25\frac{3}{4} - 18\frac{11}{12}$.
13. $32\frac{5}{12} - 18\frac{3}{4}$.
14. $40\frac{1}{2} - 20\frac{15}{16}$.
15. $48\frac{19}{20} - 36\frac{7}{10}$.

16. The prices of wheat for a week were quoted in cents as given below. Find the difference between the highest price and each of the others.

Monday	$101\frac{1}{2}$	Thursday	$103\frac{1}{5}$
Tuesday	$99\frac{3}{4}$	Friday	$104\frac{1}{4}$
Wednesday	$100\frac{5}{8}$	Saturday	$98\frac{7}{8}$

MULTIPLICATION OF MIXED NUMBERS

90. To Multiply an Integer and a Mixed Number.

How many square feet do 460 square rods equal?

Multiply: $460 \times 30\frac{1}{4}$.

$460 \times 30 = 13{,}800$

$460 \times \frac{1}{4} = 115$

$$\begin{array}{r} 4)460 \\ 30\frac{1}{4} \\ \hline 13{,}800 \\ 115 \\ \hline 13{,}915 \end{array}$$

To find the product of an integer and a mixed number, multiply the integer by the integral and fractional parts of the mixed number separately and add the products.

91. To Multiply a Mixed Number by a Mixed Number.

Multiply $5\frac{1}{3}$ by $3\frac{1}{4}$.

Solution:

$5\frac{1}{3} = \frac{16}{3}$.

$3\frac{1}{4} = \frac{13}{4}$.

$\frac{16}{3} \times \frac{13}{4} = \frac{208}{12} = 17\frac{1}{3}$.

To multiply mixed numbers, change the mixed numbers to improper fractions and multiply.

Or, multiply separately, thus:

$5\frac{1}{3} \times 3\frac{1}{4}$

$5\frac{1}{3}$

$3\frac{1}{4}$

$\frac{5}{4} + \frac{1}{12}$

$15 + \frac{3}{3}$

$15 + \frac{3}{3} + \frac{5}{4} + \frac{1}{12}$

$\frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$

$\frac{1}{4} \times 5 = \frac{5}{4}$

$3 \times \frac{1}{3} = \frac{3}{3}$

$3 \times 5 = 15.$

$$= 15 + 1 + \tfrac{1}{12} + \tfrac{1}{12}$$
$$= 16 + 1\tfrac{4}{12} = 17\tfrac{1}{3}.$$

Treat the fractions as separate units and multiply as with whole numbers; then combine the results by addition.

WRITTEN EXERCISES

92. Multiply:

1. $6\frac{1}{3} \times 3\frac{2}{5}$.
2. $25\frac{3}{4} \times 4\frac{1}{3}$.
3. $16\frac{1}{2} \times 16\frac{1}{2}$.
4. $127\frac{2}{3} \times 2\frac{3}{4}$.
5. $375\frac{1}{5} \times 5\frac{1}{5}$.
6. $347 \times \frac{1}{2} \times 3\frac{1}{3}$.

7. How many yards are there in $16\frac{1}{2}$ rods?
8. How many feet in $25\frac{1}{4}$ rods?
9. How many square yards in $10\frac{1}{2}$ square rods?
10. How many square inches in a page of a reader that is $5\frac{1}{8}$ in. by $7\frac{3}{8}$ in.?
11. Measure a page of your arithmetic and find the number of square inches in it.
12. Measure the top of your desk and find the number of square inches in it.
13. John walks $3\frac{3}{4}$ mi. in 1 hour. How far does he walk in $11\frac{5}{8}$ hours?
14. If one acre of land costs $\$215\frac{2}{3}$, what will $8\frac{2}{5}$ acres cost?
15. How many square feet in your school room floor?

DIVISION OF MIXED NUMBERS

93. To Divide a Mixed Number by an Integer or the Reverse.

Divide: $105 \div 5\frac{1}{4}$.

$$5\frac{1}{4} = \frac{21}{4}; \quad 105 \div \frac{21}{4} = 105 \times \frac{4}{21} = \frac{420}{21} = 20.$$

To divide a mixed number by an integer, or an integer by a mixed number, change the mixed number to an improper fraction and divide.

94. To Divide a Mixed Number by a Mixed Number.

$$4\frac{1}{2} \div 3\frac{1}{3} = \frac{9}{2} \div \frac{10}{3} = \frac{27}{6} \div \frac{20}{6} = \frac{27}{20} = 1\frac{7}{20}.$$

To divide mixed numbers, reduce to improper fractions and divide.

WRITTEN EXERCISES

95. Divide:

1. $8\frac{1}{4} \div 2\frac{3}{5}$.
2. $4\frac{2}{8} \div 3\frac{3}{4}$.
3. $6\frac{3}{8} \div 1\frac{1}{6}$.
4. $6 \div 2\frac{1}{2}$.
5. $5 \div 3\frac{1}{3}$.
6. $5\frac{1}{2} \div 3$.
7. $7\frac{1}{2} \div 2\frac{1}{5}$.
8. $10\frac{5}{8} \div 5\frac{1}{6}$.
9. $4 \div 2\frac{1}{5}$.

96. Solve, multiplying or dividing as necessary:

1. The conductors on the street cars of a certain city receive $22\frac{1}{2}$ cents an hour.

Alfred Harris worked as follows for one week:

Sunday $8\frac{3}{4}$ hours.
Monday $10\frac{1}{5}$ hours.
Tuesday $9\frac{1}{3}$ hours.
Wednesday $12\frac{1}{2}$ hours.
Thursday $6\frac{3}{4}$ hours.
Friday $11\frac{3}{8}$ hours.
Saturday $13\frac{1}{6}$ hours.

How much did he earn during the week?

2. How many square yards in $186\frac{2}{5}$ square rods?

3. My garden is $25\frac{3}{5}$ yards long and $16\frac{4}{7}$ yards wide. How many square rods does it contain?

4. If a block of granite weighs $4\frac{11}{16}$ tons, what is the weight of a pedestal containing 96 blocks?

DECIMAL FRACTIONS

97. A **decimal** fraction is a fraction whose denominator is 10 or some power of 10. Usually the denominator is not written but is indicated by a decimal point placed before the numerator. .06, .004, are decimal fractions.

Decimal fractions written with a decimal point are called **decimals.**

A decimal of less value than 1 is called a **pure decimal,** as .01, .25.

A decimal containing a whole number is called a **mixed decimal,** as 2.56, 3.02.

What is the first place to the right of the decimal point called?

ORAL AND WRITTEN EXERCISES

98. Read or write as directed:

1. Read: .025; .0225; .00225; 2.25; 22.5; 2.6; 26; .260; .026; .0026; .00026; 26.000; 26.26; 262.6; 2.626.

2. Write as decimals: three thousandths; one millionth; two hundred sixty-five ten-thousandths; one, and three thousand, five hundred seventy-four hundred-thousandths; six hundred, and six hundred-thousandths.

3. Is there any difference in the values of .6 and of .6000? Tell why. Write them both as common fractions.

4. Write as decimals:

$\frac{23}{100}$ $\frac{6}{100}$ $\frac{175}{1000}$ $\frac{2}{1000}$ $\frac{57}{10}$

$\frac{186}{1000}$ $\frac{145}{10}$ $\frac{987}{1000000}$ $\frac{28}{1000}$ $\frac{305}{1000}$

5. Write in decimal form:

One dollar, two cents, three mills.
One mill.
Ten dollars, one cent, one mill.
One thousand dollars, one mill.

REDUCTION OF DECIMALS

99. REDUCING DECIMALS TO COMMON FRACTIONS.

Reduce .16 to a common fraction.

$$.16 = \frac{16}{100} = \frac{4}{25}$$

To reduce a decimal to a common fraction, supply the denominator indicated and reduce the fraction to its lowest terms.

WRITTEN EXERCISES

100. Reduce to common fractions or mixed numbers:

1. .125.	8. 14.22.	15. .00144.
2. .064.	9. 37.0666.	16. 10.001.
3. .0018.	10. 24.24.	17. 304.516.
4. .750.	11. 708.096.	18. .00302.
5. .00018.	12. 367.009.	19. 20.525.
6. .0002.	13. 2.002.	20. 3.004.
7. 3.16.	14. .0375.	21. 256.175.

101. REDUCING COMMON FRACTIONS TO DECIMALS.

Reduce $\frac{4}{5}$ to a decimal.

$$5\overline{)4.0}$$
$$.8$$

Reduce $\frac{1}{2\,5}$ to a decimal.

$$25\overline{)1.00}$$
$$.04$$
$$1.00$$

To reduce a common fraction to a decimal, add as many ciphers as may be necessary to the numerator and divide by the denominator. Point off as many places as the number of ciphers added, prefixing ciphers to the quotient, if required, to make the proper number of places.

Reduce $\frac{1}{3}$ to the decimal form:

$$3\overline{)1.000}$$
$$.333\tfrac{1}{3}$$

If after three divisions a remainder is still left, it is usually better to write it over the divisor as a common fraction.

WRITTEN EXERCISES

102. Reduce to the decimal form:

1. $\frac{1}{5}$.
2. $\frac{1}{8}$.
3. $\frac{6}{7}$.
4. $\frac{3}{4}$.
5. $\frac{1}{12}$.
6. $\frac{4}{7}$.
7. $\frac{16}{20}$.
8. $\frac{4}{15}$.
9. $\frac{9}{10}$.
10. $\frac{5}{6}$.
11. $1\frac{2}{5}$.
12. $17\frac{3}{4}$.
13. $18\frac{1}{2}$.
14. $20\frac{1}{4}$.
15. $36\frac{1}{8}$.
16. $5\frac{1}{10}$.
17. $7\frac{3}{12}$.

ADDITION AND SUBTRACTION OF DECIMALS

103. How do you write decimals for addition?

1. Add:

$32.654 + 18.275 + 3.06$; $76.405 + 1.83769 + .2793$.

How do you write decimals for subtraction?

2. Subtract:

(1) $16.075 - 4.285$ (2) $7.306 - .042$

(3) $16.04 - 1.604$. (4) $.023 - .0023$.

3. One million dollars ten cents, from one million, one thousand dollars, ten cents, one mill.

WRITTEN EXERCISE

104. Solve:

1. A room is 21.5 ft. long, 16.2 ft. wide. What length of baseboard will it require?

2. A man owns 3 city plots containing 1.413 acres, 2.04 acres, and 6.125 acres. He sold $5\frac{1}{2}$ acres. How many acres had he left?

3. A corn plant grew .1 in. on Monday, .003 in. on Tuesday, $\frac{1}{2}$ in. on Wednesday, and on Thursday as much as on the three preceding days. How much did it grow in the 4 days?

4. For each of the following commodities find how much the highest price exceeded each of its other prices:

	Jan. 7, 1908	July 7, 1908	Aug. 7, 1908
Wheat, No. 2 red	1.13½	1.45	1.03¾
Corn, No. 2 mixed	.80	.78	.88
Oats, No. 2 white	.50½	.57½	.59
Flour, Minn. Patent	6.25	6.60	6.00
Lard, choice	11.90	12.15	9.85
Pork, mess	22.00	21.00	17.50
Beef, family	14.00	13.74	17.25
Coffee, No. 7 Rio	.06⅜	.07⅜	.06½
Sugar, granulated	4.85	4.75	5.20
Steel billets	24.00	23.00	25.00
Lead	4.27½	4.37½	4.60
Tin	29.27½	28.85	30.80

MULTIPLICATION OF DECIMALS

105. Multiply $1 \times \frac{1}{10}$; $\frac{1}{10} \times \frac{1}{10}$; $\frac{1}{10} \times \frac{1}{100}$.

How do the ciphers in each product compare in number with those in the two factors together? Explain.

Write each of the above multipliers, multiplicands, and products as a decimal.

1. $.1 \times 1 = .1$ ($1 \times \frac{1}{10}$). One-tenth taken once = .1.

2. $.1 \times .1 = .01$ ($\frac{1}{10}$ of $\frac{1}{10}$). One-tenth taken one-tenth times = .01.

3. $.01 \times 1 = .01$ ($\frac{1}{100}$ of 1). One-hundredth taken once = .01.

4. $.01 \times .1 = .001$ ($\frac{1}{100} \times \frac{1}{10}$). One-hundredth taken one-tenth times = .001.

In eacn of the above problems how many decimal places are there in the multiplier? In the multiplicand? In both? In the product?

106. The product of two numbers containing decimals contains as many decimal places as both of the numbers. Explain why.

46.385×2.56

SOLUTION:

$$\begin{array}{r} 46.385 \\ 2.56 \\ \hline 278310 \\ 231925 \\ 92770 \\ \hline 118.74560 \end{array}$$

To multiply decimals, multiply as with whole numbers. In the product, point off as many places at the right as there are decimal places in both multiplicand and multiplier.

$$\begin{array}{r} .0256 \\ .047 \\ \hline 1792 \\ 1024 \\ \hline .0012032 \end{array}$$

In this example the number of figures in the product is two less than the number of decimal places of both multiplicand and multiplier; therefore, two ciphers are *prefixed* to the product. Why are they not placed *after* the product.

WRITTEN EXERCISES

107. Find the product of the following:

1. 2.25×5.	**6.** $.0019 \times .91$.	**11.** $.375 \times 23\frac{3}{8}$.
2. 35.04×7.3.	**7.** $475 \times .475$.	**12.** $62\frac{1}{2} \times .15$.
3. $.038 \times 46$.	**8.** $.0214 \times .078$.	**13.** $43\frac{5}{8} \times 6.25$.
4. 150.85×4.013.	**9.** $2200 \times .0022$.	**14.** $.875 \times \frac{3}{4}$.
5. 8.04×408.	**10.** $35\frac{3}{4} \times .125$.	**15.** $12.435 \times \frac{7}{8}$.

16. What will .016 of an acre of land cost at \$400 per acre?

17. If lumber is worth \$40 a thousand, how much will 125 feet cost?

18. A train goes 20.25 mi. in one hour. How far will it go in $3\frac{3}{4}$ hours? In 4.05 hours? In 5.001 hours? In $6\frac{1}{2}$ hours?

BILLS

108. The following is a bill receipted:

MARSHALL FIELD & COMPANY.

Importers, Retailers, Manufacturers,
State, Washington, Randolph, & Wabash Sts.,

CHICAGO, ILL., July 8, 1909.

Sold to
MR. JOHN DOE,
1017 State St.

July	6	5 yd. silk,	.95	4	75		
		4 spools thread,	.04		16		
	13	2 pair rubbers,	.55	1	10		
		6 cakes soap,	.08		48		
	24	1 umbrella,		1	48	7	97
		Received payment, MARSHALL FIELD & Co.					

Make out and receipt 3 bills, selecting articles and using prices named in the following lists, and supplying the names of the parties to the transactions:

Sugar	@	$ .06½	Eggs	@	$ .23
Coffee	"	.32	Milk	"	.06
Slippers	"	1.50	Lace	"	2.25
Raisins	"	.12½	Muslin	"	.07
Silk	"	1.75	Crackers	"	.08
Calico	"	.10	Molasses	"	.80
Hose	"	.25	Oil	"	.14
Ribbon	"	.35	Codfish	"	.09
Gloves	"	1.25	Tea	"	.60
Collars	"	.15	Rice	"	.08
Shoes	"	3.50	Starch	"	.05

DIVISION OF DECIMALS

109. DIVIDING DECIMALS OF THE SAME DENOMINATION.

$\frac{25}{100} \div \frac{5}{100} = 5$. The denominators being the same, how do we divide common fractions?

$.25 \div .05 = \frac{25}{100} \div \frac{5}{100} = 5.$

$\frac{4}{10} \div \frac{4}{10} = (.4 \div .4) = 1.$

$.075 \div .025 = \frac{75}{1000} \div \frac{25}{1000} = 3.$

If a decimal is divided by a decimal of the same denomination no larger than itself, the quotient will be a whole or a mixed number. Explain why.

110. DIVIDING DECIMALS WHEN THERE ARE MORE PLACES IN THE DIVISOR THAN IN THE DIVIDEND.

Divide:

$25 \div .5 = 50.$

SOLUTION: $25 = \frac{250}{10}$; $.5 = \frac{5}{10}$.

$\frac{250}{10} \div \frac{5}{10} = 250 \div 5 = 50.$

$$.5\overline{)25.0} \quad 50$$

Divide:

$2.5 \div .005 = 500.$

SOLUTION: $\frac{2500}{1000} \div \frac{5}{1000} = 2500 \div 5 = 500.$

$$.005\overline{)2.500} \quad 500$$

Divide:

$.2 \div .08.$

SOLUTION: $\frac{20}{100} \div \frac{8}{100} = \frac{20}{8} = 2\frac{1}{2} = 2.5.$

$$.08\overline{).200} \quad 2.5$$

16

40

In dividing decimals, when the divisor contains more decimal places than the dividend, annex enough ciphers to the dividend to make up the difference.

Why are the ciphers not prefixed? What will be the nature of the quotient?

111. DIVIDING DECIMALS WHEN THERE ARE MORE PLACES IN THE DIVIDEND THAN IN THE DIVISOR.

In dividing common fractions having different denominators, how do you proceed?

(Remember that inverting the divisor is a short method of reducing to a common denominator; see page 42.)

Reduce to a common denominator, $\frac{5}{10}$, $\frac{2}{100}$, $\frac{3}{1000}$, 4.

Reduce to the same denomination, .5, .02, .003, 4.

In changing several decimal fractions to the same denomination, which of the fractions will determine the new denomination? Why?

Reduce the fractions in each of the following problems to a common denominator and divide the numerators.

Divide $\frac{1}{100}$ by $\frac{1}{10}$; $\frac{1}{10}$ by 1; $\frac{1}{1000}$ by $\frac{1}{100}$; $\frac{1}{1000}$ by $\frac{1}{10}$.

Can you by observing the number of ciphers in the denominators of the dividend and the divisor tell how many that of the quotient will have? Explain how.

112. Divide .01 by .1; .1 by 1; .001 by .01; .001 by .1.

In the above problems can you tell by observing the dividend and the divisor how many decimal places the quotient will have? Explain how.

Divide: $2.5 \div 5 = .5$.

What is the common denominator?

$$2.5 = 2\tfrac{5}{10} = \tfrac{25}{10}; \quad 5 = \tfrac{50}{10};$$

$$\tfrac{25}{10} \div \tfrac{50}{10} = \tfrac{25}{50} = \tfrac{1}{2} = \tfrac{5}{10} = .5.$$

Divide: $2.5 \div .5 = 5$.

What is the common denominator?

$$\tfrac{25}{10} \div \tfrac{5}{10} = 5. \qquad .5\overline{)2.5}\ 5$$

Divide: $.25 \div .5 = .5$.

$$\tfrac{25}{100} \div \tfrac{50}{100} = \tfrac{25}{50} = \tfrac{5}{10} = .5 \qquad .5\overline{).25}\ .5$$

Divide: $.16 \div 8$.

SOLUTION: $\frac{16}{100} \div \frac{800}{100} = \frac{16}{800} = \frac{2}{100} = .02$.

$$8\overline{).16}\quad .02$$

Divide: $.004 \div .2$; $6.25 \div 2.5$; $.0012 \div .03$.

When dividing decimals, if there are more decimal places in the dividend than in the divisor, the number of decimal places in the quotient will equal their difference.

If more places must be pointed off than the number of figures in the quotient, *prefix* enough ciphers to equal the difference.

113. A SHORT METHOD OF DIVIDING DECIMALS:

$$62.5 \times 10 = 625. \qquad 6.25 \times 100 = 625.$$

How do you multiply a decimal by 10? By 100? By 1000?

Does multiplying both numerator and denominator of a fraction alter its value?

$$\frac{3.25}{.25} \times \frac{100}{100} = \frac{325}{25}.$$

$$\frac{47.5}{.25} \times \frac{100}{100} = \frac{4750}{25}.$$

The method given above may be used to advantage in all cases of long division, involving decimals. For example,

Divide 3.08654 by 2.15.

SOLUTION: $\frac{3.08654}{2.15} \times \frac{100}{100} = \frac{308.654}{215}$.

$$215\overline{)308.654}\quad 1.435$$

215
936
860
765
645
1204
1075
129

114. In each of the following problems, by what do you multiply the divisor and the dividend to make a whole number of the divisor?

$4368 \div .104$; $34.28 \div 11.6$; $.3872 \div .236$; $441.375 \div 26.05$.

Before dividing move the decimal point of both dividend and divisor to the right enough places to make the divisor a whole number.

Place the decimal point of the quotient over that of the dividend and divide, prefixing ciphers to the quotient if necessary.

If there are more decimal places in the divisor than in the dividend, add enough ciphers to the dividend to equal the difference. If, after adding several ciphers and dividing, there is still a remainder, place it over the divisor, making a common fraction as in dividing whole numbers.

WRITTEN EXERCISES

115. Solve:

1. $327.56 \div .233$.
2. $417 \div 25$.
3. $16.893 \div .16893$.
4. $2.727 \div 272.7$.
5. $6000 \div .25$.
6. $\$487.60 \div .15$.
7. $24 \div .012$.

8. A lot containing 5,000 sq. ft. is $233.33\frac{1}{3}$ ft. long. How wide is it?

9. A room is $26.66\frac{1}{3}$ ft. long; its area is 780 sq. ft. What is its width?

10. The area of a circle $= 3.1416 \times$ the square of the radius (πr^2). A certain sundial has an area of 12.5664 ft. What is the length of its radius?

11. Sound travels through the air at the rate of 1134 ft. a second. If a shot is fired 234.25 ft. distant from you, in what part of a second will the sound reach you?

12. In one square rod there are 30.25 square yards. How many square rods are there in 3586.375 square yards?

13. Divide \$405.25 equally among 5 girls. How much does each receive?

REVIEW

116. Questions on Subject Matter

1. What is a simple fraction? Give an example.

2. When is a fraction in its lowest terms?

3. What does the numerator of a fraction show? What does the denominator show?

4. What is the general meaning of the word *reduction?*

5. What does reduction mean as used in arithmetic?

6. Why must fractions be reduced to a common denominator before they can be added?

7. In how many ways can the division of 3 by 4 be indicated?

8. Which of the following fractions are decimal fractions: $\frac{4}{10}$, $\frac{3}{4}$, .04, $\frac{5}{200}$, $\frac{7}{1000}$, .033, 2.8, $4\frac{3}{4}$?

9. What is the denominator of a decimal?

10. What is the numerator of a decimal?

11. What is meant by a unit?

12. What is meant by the unit of a fraction?

13. How may a complex fraction be reduced to a simple one?

14. How do you reduce a decimal to a common fraction?

15. Show that the process is a real case of reduction.

16. How would you multiply $\frac{3}{4}$ by $\frac{5}{7}$? What two things are required to be done?

17. Since division is the converse of multiplication, how could you divide $\frac{15}{28}$ by $\frac{3}{4}$? By $\frac{5}{7}$?

18. Show that the decimal form is preferable to the fraction form in the following problem:

$$.008 \times 2.04 = \text{what?}$$

$$\frac{8}{1000} \times 2\frac{4}{100} = \text{what?}$$

WRITTEN EXERCISES

117. Solve:

1. Reduce to lowest terms:

$$\frac{15}{75},\ \frac{48}{80},\ \frac{24}{30},\ \frac{40}{60},\ \frac{42}{56},\ \frac{64}{144}.$$

2. Find the sum of:

$$\frac{6}{11},\ \frac{7}{12},\ \frac{9}{14},\ \frac{5}{16}.$$

3. Find the difference of $\frac{5}{6} - \frac{11}{12}$.

4. Fred picked half a bushel of peas. He sold them at $\$.11\frac{1}{2}$ a quart. How much did he get for them?

5. What will be the cost of $3\frac{3}{4}$ doz. oranges at $\$\frac{80}{20}$ a dozen?

6. $\frac{3}{4}$ of the value of a house was \$1248. What was the whole value of the house?

7. A man bought a cow for \$48.50. The cow cost him $\frac{2}{5}$ as much as he paid for a horse. How much did he pay for the horse?

8. A room is $24\frac{3}{4}$ ft. wide. How many strips of carpeting $\frac{3}{4}$ yd. wide will be required to cover it?

9. A sugar dealer had $15\frac{3}{8}$ gal. of maple syrup which he put up in bottles, each holding $\frac{3}{8}$ of a gallon. How many bottles did he have?

10. He sold the syrup at $\$\frac{5}{8}$ a bottle. He had paid $\$6\frac{1}{2}$ for it. How much did he gain?

11. The circumference of a circle is 3.1416 times the diameter. Find the circumference of a circle whose radius is 42.15 ft.

12. A real estate dealer bought 1013.88 acres of land which he divided into lots of equal size. There were 426 in all. How many acres were there in each lot?

13. A man left $\frac{2}{7}$, $\frac{1}{5}$, and $\frac{2}{15}$ of his money to three different

charity organizations. The total amount of property that he left was $21,105. How much did he leave to each?

14. A tank of oil is $\frac{4}{5}$ full. If $\frac{2}{3}$ of the contents of the tank are drawn off and then $\frac{5}{6}$ of the remainder, what part is left?

15. From a pile of apples containing $368\frac{4}{5}$ bu., $72\frac{6}{7}$ bu. were sold. How many remained?

16. Divide 6.26 by .00025; 3430 by .049.

17. A rectangular field is $33\frac{3}{4}$ rods long and $15\frac{5}{6}$ rods wide. How many square rods does it contain?

18. A farmer had 650 bushels of corn. He sold $\frac{5}{13}$ of them. How many did he sell? How many had he left?

19. From his income a man saves $250 a year. He spends $\frac{5}{8}$ of his income for household expenses, $\frac{5}{24}$ of it for clothing, and $\frac{1}{12}$ of it for his vacation expenses. What is his income?

20. An automobile went 275.16 miles in 10 hours. How far did it travel in one hour?

21. The area of a rectangle is 3798.75 sq. ft. One side of the rectangle measures 100 feet. What are the lengths of the other sides?

22. A dealer bought 100 pictures for which he paid $965. What was the cost of one picture?

SUMMARY OF CHAPTER II

COMMON FRACTIONS.

Reduction.
Fundamental processes.
Simple Fractions.
Mixed Numbers.

DECIMAL FRACTIONS.

Reduction.
Fundamental processes.

CHAPTER III

DENOMINATE NUMBERS — APPLICATIONS OF DENOMINATE NUMBERS

Simple and Compound

118. What is an abstract number? A concrete number? A unit?

Name some unit of measurement.

Concrete numbers naming units of measurement fixed by law are called **denominate numbers.**

Denominate numbers of a single denomination, as 2 pounds, are called **simple denominate numbers**; those of more than one denomination, as 2 lb. 3 oz., are called **compound denominate numbers.**

For the tables of denominate numbers, see p. 264.

REDUCTION

119. Reducing to Lower Denominations.

How many pint bottles can be filled from a can of milk containing 3 gal. 3 qt. 1 pt.?

Suggestion: Gal. 3×4 (qt. in gal.) $= 12$ qt.

Qt. $3 + 12 = 15$; 15×2 (pt. in qt.) $= 30$ pt.

Pt. $1 + 30 = 31$ pt.

Reducing a denominate number to lower denominations is called **reduction descending.**

To reduce a compound denominate number to its lowest denomination, multiply the number of units of each denomination, beginning with the highest, by the number representing the ratio of its unit to that of the next lower denomination, and add to the product in each case the units, if any, of the lower denomination.

120. Reduce to lower denominations:

1. $1, 5 dimes 6 cts.
2. 16 gal. 3 qt. 1 pt. 1 gi.
3. 1 T. 3 cwt. 17 lb. 9 oz.
4. 12 cwt. 3 gr.
5. 1 lb. 10 ℥ 7 ʒ 2 ℈ 15 gr.
6. 1 mi. 150 rd. 3 yd.
7. 4 yd. 2 ft. 10 in.
8. 2 mi. 75 ch. 80 li.
9. 3 sq. mi. 500 A. 15 sq. rd. 26 sq. yd. 8 sq. ft. 120 sq. in.
10. 5 sq. mi. 320 A. 5 sq. ch. 12 sq. rd. 315 sq. li.
11. 10 cu. yd. 25 cu. ft. 1600 cu. in.
12. 3 bundles of paper, 1 ream, 15 quires, 20 sheets.
13. 10 gr. gross, 8 gross, 9 doz.
14. How many years are 3 score and 10?
15. 1 yr. 4 mo. 2 wk. 6 da.

121. REDUCING TO HIGHER DENOMINATIONS.

In 24 pt. how many gallons?

$$\begin{array}{r} 2)\underline{24}\text{ pt.} \\ 4)\underline{12}\text{ qt.} \\ 3\text{ gal.} \end{array}$$

Reduce 6500 lb. to higher denominations.

SOLUTION:

$$\begin{array}{r} 100)\underline{6550}\text{ lb.} \\ 20)\underline{65}\text{ cwt. } 50\text{ lb.} \\ 3\text{ T. } 5\text{ cwt.} \end{array}$$

Ans. 3 T. 5 cwt. 50 lb.

Changing denominate numbers to higher denominations is called **reduction ascending.**

To reduce a denominate number to one of higher denominations, divide the number of units given by the ratio of a unit of the next higher denomination to one of the given denomination. The remainder, if any, will be of the lower denomination. Divide each succeeding quotient in like manner.

WRITTEN EXERCISES

122. Reduce:

400 pt. (dry) to bushels.
850 lb. to hundredweight.
672 cwt. to tons.
5267 sec. to hours.
139,968 cu. in. to cubic yards.
1467 in. to rods.
800 rd. to miles.
245 hr. to days.
122 sq. ft. to square yards.
520 sq. rd. to acres.

ORAL EXERCISES

123. How many inches in 4 yd. 2 ft. 8 in.?

How many pounds in 64 ounces?

How many pints in 2 bu. 3 pk. 1 qt.?

How many sq. in. in 3 sq. yd.?

How many ounces in 5 cwt.?

WRITTEN EXERCISES

124. Solve:

1. Reduce to lower denominations:

(1) 30 sq. yd. 8 sq. ft. 56 sq. in.
(2) 25 " " 6 " " 100 " "
(3) 18 " " 5 " " 75 " "

2. Mr. Morse sold 5 bu. 3 pk. 6 qt. of chestnuts for 15 ct. a quart. How much did he receive for them?

3. Mr. Dunlap bought $12\frac{1}{2}$ bu. of potatoes for \$12.50 and sold them for \$.35 a peck. How much did he make?

4. Mr. Harris sold $31\frac{1}{2}$ gal. of molasses at $\$0.12\frac{1}{2}$ a quart. How much did he receive for it?

5. A milk dealer sold to Mr. Carter 12 gal. 3 qt. of milk; to Mr. Hammond 15 gal. 2 qt. 1 pt.; to Mr. Francis 25 gal. The purchasers were grocers and sold the milk to their customers for $0.04 a pint. How much did each grocer receive for his milk?

6. What is the value of 150 gal. of milk at $0.03 a pint?

7. Change 327 pints to quarts and gallons and prove the result.

8. How many cords in a pile of wood 4 ft. wide, 8 ft. high, 120 ft. long?

9. A body immersed in water loses in weight as much as a body of water of equal size weighs. How much will a cubic foot of lead weigh in water?

1 cubic foot of water weighs 62.5 lb.
1 cubic foot of lead weighs in the air 709.5 lb.

10. A diver examined a sunken vessel $6\frac{1}{2}$ fathoms under water. How many feet under water was it?

11. He brought up a treasure chest 2 ft. × 1 ft. × 1 ft., that weighed, on land, including contents, 200 lb. How much did it weigh in the water? How many cubic feet in the chest?

12. The steel boat *Cupid* weighs 2400 lb. When empty, it displaces 880 cu. ft. of water; when fully loaded, 2100 cu. ft. How many pounds of load can be put on the boat before sinking it?

A body will float if it weighs no more than a body of water of equal size.

13. The steamship *Norma* weighs about 3800 tons; her machinery, 1310 tons; her fuel for a trip across the ocean, 1500 tons. When loaded she weighs 9550 tons. What is her load? How many cubic feet of water does she displace when loaded?

125. REDUCING TO DECIMALS OR COMMON FRACTIONS OF HIGHER DENOMINATIONS.

1. Reduce 5 bu. 2 pk. 8 qt. to mixed decimals of a bushel.

SOLUTION: 1 bu. = 32 qts.
8 qts. = $\frac{1}{4}$ = .25 of a bushel.
1 bu. = 4 pk.
2 pk. = $\frac{1}{2}$ = .5 of a bushel.
5 + .25 + .5 = 5.75 bu.

2. Reduce 20 rd. 4 yd. 2 ft. 3 in. to mixed decimals of a rod.

SOLUTION: 3 in. = $\frac{1}{4}$ ft. = .25 ft.

3)2.25 ft.
.75 yd.

5.5)4.7500(.863 +
4 40
350
330
200
165

NOTE. Carry to three places.

Ans. 20.863+ rd.

3. Reduce 4 gal. 2 qt. 1 pt. to gallons and decimals of a gallon. Change the result to common mixed numbers.

4. Reduce 4 da. 2 hr. 40 min. to a decimal of a week.

5. What fraction of a ton is 15 cwt. 75 lb.?

In the ordinary transactions of life the use of a common fraction, or mixed number, of a single unit, is much more common than the use of compound numbers. We say, $\frac{1}{2}$ lb. of coffee, not 8 oz.; a yard and a half of ribbon, not 1 yd. 1 ft. 6 in.

ORAL EXERCISES

126. Reduce to both common and decimal fractions:

1. 4 oz.
2. 1 lb. 8 oz.
3. 5 yd. 9 in.
4. $1, 25 ct.
5. 10 mi. 160 rd.
6. 17 rd. 5 ft.
7. 1 sq. mi. 160 A.
8. 6 bu. 3 pk.
9. 5 gal. 1 qt.

ADDITION AND SUBTRACTION

127. Add:

Yd.	Ft.	In.	
15	2	9	
8	4	16	
23	6	25	= 2 ft. 1 in.
2	2		
	8	ft.	= 2 yd.

25 yd. = 4 rd. 3 yd.

The sum = 4 rd. 3 yd. 2 ft. 1 in.

128. Subtract:

Yr.	Mo.	Da.	
1910	7	16	We "borrow" from 7.
1907	6	19	16 da. + 30 da. = 46 da.
3	0	27	46 da. − 19 da. = 27 da.

This method is seldom employed except in finding the difference between two dates.

In both addition and subtraction it is usually better, before adding or subtracting, to reduce compound numbers to decimals or to common fractions of the highest denomination, or to a whole number of a lower denomination.

Subtract: 6 yd. 2 ft. 6 in. − 4 yd. 1 ft. 6 in.

SOLUTION: 6 yd. 2 ft. 6 in. = $6\frac{5}{6}$ yd. or 246 in.

4 yd. 1 ft. 6 in. = $4\frac{3}{6}$ yd. 162 in.

$2\frac{2}{6}$ 84 in.

WRITTEN EXERCISES

129. Solve:

1. Mr. Alford, in taking an account of his stock, found the following remnants of ribbon:

10 yd. 1 ft. 6 in.;	5 yd. 9 in.;
7 yd. 2 ft. 3 in.;	2 yd. 1 ft. 6 in.

He sold the lot to a dealer in remnants at 20 cents a yard. What did he receive for it?

2. The purchaser sold the following pieces to girls, dressing dolls for a fair, at 25 cents a yard:

2 yd. 6 in.;	5 yd. 1 ft.;
4 yd. 1 ft. 3 in.;	3 yd. 9 in.

How much did he receive for them?

3. How much did the dealer have left after making the above sales?

4. George Washington was born Feb. 22, 1732. He died Dec. 14, 1799. How old was he at his death?

5. Exactly how long ago to-day was the Declaration of Independence signed?

6. Exactly how old are you to-day?

7. Dewey captured Manila May 1, 1898. How long ago was that?

8. Mr. Smith has 2 fields, one containing 7 A. 9 sq. rd. 3 sq. yd. 2 sq. ft. of land, and the other 5 A. 121 sq. rd. 4 sq. ft. How much larger is the first than the second?

9. A farmer sold to one man 1 T. 2 cwt. 40 lb. of hay, to another 4 T. 70 lb., to the third 2 T. 15 cwt. 10 lb., and to the fourth 5 T. 13 cwt. 23 lb. How much did he sell to all?

10. In the month of March a grocer sold 125 gal. 2 qt. 2 gi. of kerosene, but in August he sold only 52 gal. 1 pt. 3 gi. How much more did he sell in March than in August?

MULTIPLICATION

130. How much oil will three cans hold, if each holds 2 gal. 3 qt. 1 pt.?

Gal.	Qt.	Pt.
2	3	1
		3

SOLUTION:

3×1 pt. $= 3$ pt. $= 1$ qt. 1 pt.

3×3 qt. $= 9$ qt.

9 qt. $+ 1$ qt. $= 10$ qt. $= 2$ gal. 2 qt.

3×2 gal. $= 6$ gal.

6 gal. $+ 2$ gal. $= 8$ gal.

Ans. 8 gal. 2 qt. 1 pt.

Another method is to reduce to common fractions of the highest denomination before multiplying.

SOLUTION:

1 pt. $= \frac{1}{8}$ gal.

3 qt. $= \frac{3}{4}$ gal.

$\frac{3}{4} + \frac{1}{8} = \frac{7}{8}$

$2\frac{7}{8}$ gal. $\times 3 = 8\frac{5}{8}$ gal.

Still another method is to reduce to decimal fractions of the highest denomination before multiplying.

SOLUTION:

1 pt. $= .125$ gal.

3 qt. $= .75$ gal.

2.875 gal. $\times 3$

8.625 gal. $= 8\frac{5}{8}$ gal.

$= 8$ gal. 2 qt. 1 pt.

Which is the easiest way?

In some problems the reduction to a decimal or common fraction may be too complicated. In such a case it is some-

times easier to reduce the compound number to its lowest denomination; thus:

131. Multiply 1 rd. 3 yd. 2 ft. by 16.

SOLUTION:

$$\begin{aligned} 1 \text{ rd.} &= 16\tfrac{1}{2} \text{ ft.} \\ 3 \text{ yd.} &= \ 9 \text{ ft.} \\ &\quad\ \underline{2 \text{ ft.}} \\ &\quad\ 27.5 \text{ ft.} \times 16 = 440 \text{ ft.} \end{aligned}$$

WRITTEN EXERCISES

132. Multiply (using the simplest method):

1. A field is 20 rd. 4 yd. 2 ft. 5 in. long. How far must a plowman walk in plowing 25 furrows the long way of the field?

2. The "mean" solar year is 365 da. 5 hr. 48 min. 46 sec. exactly. How long a time does one live in 15 "mean" solar years?

3. How many square feet in a table 4 ft. 6 in. × 8 ft. 4 in.? Which method do you employ in this example?

4. What is the volume of a box 2 ft. 6 in. × 3 ft. 2 in. × 4 ft. 8 in.?

5. An automobile tank holds 9 gal. 2 qt. 1 pt. of gasoline. How much gasoline will it take to fill it once each day for a week?

6. What will be the actual weight in pounds of 12 pound-packages of cereal, if each falls $\frac{1}{2}$ oz. short in weight?

7. A real estate dealer divided a plot of ground into 25 lots, each containing 2500 sq. ft. How many acres did the plot contain?

8. A man spent 2 hr. 25 min. 45 sec. on the train going to business each day. Find the time that he so spent in February, 1908.

DIVISION

133. The committee have 20 lb. 10 oz. of candy, with which to fill 66 bags for the Christmas tree. How much can be put into each bag?

Divide 20 lb. 10 oz. by 66.

SOLUTION: 20 lb. 10 oz. = 330 oz.
330 oz. ÷ 66 = 5 oz.

Divide: 8 gal. 3 qt. by 5.

SOLUTION: 8 gal. 3 qt. = 8.75 qt.
8.75 ÷ 5 = 1.75.

Divide: 14 bu. 3 pk. 7 qt. 1 pt. by 4.

SOLUTION:

	Bu.	Pk.	Qt.	Pt.
4)14	3	2	7	1
	3	2	7	$1\frac{2}{4}$

14 bu. ÷ 4 = 3 bu. and 2 remainder.
2 bu. = 8 pk.; 3 pk. + 8 pk. = 11 pk.
11 pk. ÷ 4 = 2 pk. + 3 remainder.
3 pk. = 24 qt.; 24 qt. + 7 qt. = 31 qt.
31 qt. ÷ 4 = 7 qt. + 3 remainder.
3 qt. = 6 pt.; 6 pt. + 1 pt. = 7 pt.
7 pt. ÷ 4 = $1\frac{3}{4}$ pt.

Which of the methods is the simplest?

Divide, using first one and then another method:

15 yd. 2 ft. 11 in. ÷ 6.
25 mi. 75 rd. 1 ft. ÷ 8.

To divide a compound number by an abstract number, divide the number of each denomination in turn by the divisor. Reduce the remainder, if there is one, to the next lower denomination and add it to the number of that denomination before dividing. Or,

Reduce the dividend to a simple denominate number and divide. Or,

If the divisor is a compound number, reduce both terms to the same denomination, regard the numbers as abstract, and divide.

WRITTEN EXERCISES

134. Solve:

1. 25 bu. 3 pk. 6 qt. 1 pt. ÷ 5.
2. 46 yd. 2 ft. 11 in. ÷ 7.
3. 60 T. 1600 lb. 12 oz. ÷ 11.
4. 12 dollars, 6 dimes, 9 cents ÷ 5.
5. 48 gal. 3 qt. 1 pt. ÷ 10.
6. A man purchased land which faced on Green Street for a distance of 40 rods and 15 feet. He divided this distance into 10 equal parts. What was the width of each lot?
7. A lot 6 rd. 1 ft. × 5 rd. 10 ft. is to be laid out in tennis courts. How many square feet does it contain? How many courts 78 × 36 ft. can be made in it? How much land will be left on the sides? the ends?
8. It is to be fenced with 2 strips of poultry wire, each 4 ft. wide. How many linear feet of wire will it take? How many square feet?
9. The wire is supported on posts 10 ft. apart. How many posts will be required? (Draw an illustration.)
10. The poultry wire costs $\frac{1}{2}$ cent a square foot. What is the total cost?
11. The posts are 4 in. square and are set 3 ft. in the ground. At 6 cents a foot, what is their cost?
12. Into how many fields, each containing $10\frac{1}{2}$ acres, can a farmer divide his farm of $157\frac{1}{2}$ acres?
13. A grocer bought 3 barrels of apples and sold them by the basket, each basket containing 1.5 pk. How many baskets of apples did he sell?
14. A carload of coal containing 86,000 lb. was put into bins that held 10 T. 15 cwt. each. How many bins were filled from the carload?

MEASUREMENT OF PLANE SURFACES

135. To avoid misunderstanding and dishonesty all governments fix **standard measures.** To these standards all measures in use in business transactions must conform. The standard is usually a single measure of a kind, as a yard for length, a bushel and a gallon for capacity, a pound for weight. All other measures used must bear a certain ratio to the standard.

Not only do governments fix standards of measurements, but they pass laws making it a crime to use false measures, and have provided for the punishment of those doing so.

RECTANGLES AND TRAPEZOIDS

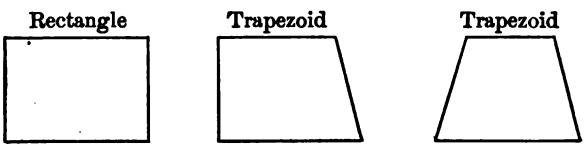


FIG. 1. FIG. 2. FIG. 3.

136. What is the difference between Fig. 2 and Fig. 1? Between Fig. 3 and Fig. 1? Between Fig. 2 and Fig. 3? What is Fig. 2 called?

Of what kind of lines is it made?

Lines that have the same direction are said to be **parallel.**

A quadrangle having its opposite sides parallel is called a **parallelogram.**

A **rectangle** is a plane figure bounded by four straight lines, its opposite sides being parallel, and its corners forming right angles.

A **trapezoid** is a plane figure bounded by four straight lines, only one pair of sides being parallel.

137. 1. Draw a trapezoid whose parallel sides are 4 in. and 6 in., the distance between them being 2 in. Cut out the trapezoid.

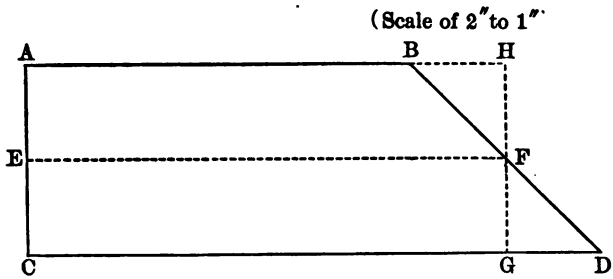
2. Bisect the vertical side AC, and draw from E, the point of bisection, a line EF parallel to the base.

3. From the point F, where this line touches the oblique line, draw a line FG, perpendicular to the base CD.

4. Fold over the triangle FGD thus formed; then cut to point of intersection F, on line FG.

5. Turn F as on a pivot, until triangle FGD takes the position FHB.

6. What kind of figure is now formed?



7. What is its length? 8. Its width? 9. Its area?

10. What is the average length of its parallel sides?

11. How does this compare with the length of the rectangle AHGC that was made from the trapezoid?

The average length of the parallel sides of a trapezoid is equal to the length of a rectangle of the same size.

The area of a trapezoid is equal to the area of a rectangle of equal altitude, whose length is the average of the two parallel sides.

To find the area of a trapezoid, find the average length of the two parallel sides and multiply this by the altitude.

ORAL EXERCISES

138. What is the area of a trapezoid whose parallel sides are:

1. 6 in. and 10 in., distance between them 4 in.?
2. 8 in. and 12 in., distance between them 6 in.?
3. 5 in. and 17 in., distance between them 10 in.?
4. 7 in. and 11 in., distance between them 7 in.?
5. 12 in. and 20 in., the distance between them 11 in.?

WRITTEN EXERCISES

139. Solve:

1. Draw a figure 2 inches by 1 inch. If it is drawn on a scale of 1 inch to 25 feet, what is the perimeter of the rectangle that it represents?

2. Find the perimeter of the rectangle represented by the figure on page 75, if drawn on a scale of $\frac{1}{2}$ inch to 25 feet.

3. Using the scale of Exercise **2**, find the number of square feet the figure represents. Of square yards.

4. Measure the area of your schoolroom and draw a plan of it on the scale of $\frac{1}{4}$ in. to 1 ft.

5. Measure the surface of your desk and draw in the proper places on your plan all the desks, using the same scale.

6. Draw to scale a trapezoid whose parallel sides are 16 in. and 64 in., and the distance between them 32 in. Indicate by dotted lines the process of finding the area.

7. Find the area of the trapezoid you have drawn.

8. Mr. Scott had a farm with the parallel sides respectively 180 rods and 150 rods and the distance between them 60 rods. (Draw to scale 1 inch = 60 rods.)

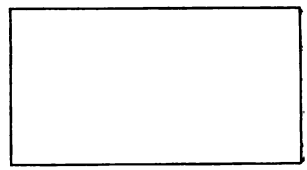
9. How many acres are there in the Scott farm?

10. Find the area of an isosceles triangle whose base is 71 in. and altitude 30 in.

WRITTEN EXERCISES

140. Solve:

1. A rectangular field is four times as long as it is wide, and contains 400 sq. rd. If it is 10 rd. wide, how long is it? What is its perimeter? How many acres does it contain?



400 sq. rd.

2. A man took a contract for making a concrete walk at \$.10 a square foot. If the walk was 10 rd. long and 5 ft. wide, how much did he receive?

3. A man built a 6-foot walk around a square plot 100 ft. on a side. What was the length of the walk inside? What was the length of the walk outside?

Make a drawing of the plot and the walk.

4. In the government survey of public lands, a square mile is called a section. The square miles are divided into 4 parts, called quarter sections. These are named from their position in the section, as, northeast quarter, or N.E. $\frac{1}{4}$, southeast quarter, or S.E. $\frac{1}{4}$, etc. Each quarter section contains 160 acres.

Draw a square to represent a square mile, and locate each of the quarter sections.

5. How many acres are there in a section? In $\frac{1}{4}$ of a quarter section? In $\frac{1}{2}$ of a quarter section?

6. A stockman purchased $10\frac{1}{2}$ sections of land for $\$3\frac{1}{2}$ an acre. What was the total cost?

7. A garden is 30 rd. long and 15 rd. wide. A man builds a fence around it 4 ft. high with posts 1 rd. apart. The posts cost \$.20 each, and the other material and labor cost \$1 per rod. How many posts will be required? What will be the total cost of the fence?

LONGITUDE AND TIME

141. The exact location of any place on the globe may be described by telling its distance north or south and east or west from other places that are known. For convenience all measurements are made to start from two lines imagined to be drawn around the earth at right angles to each other. The east and west line equally distant from the two poles is the **equator**.

Distance north or south of the equator is called **latitude**.

Imagine a circle drawn, through the place where you are standing, around the world through the north and south poles. Such a circle is called a **meridian**. You can imagine a meridian drawn through any place on the globe; so every place has its own meridian.

Distance east or west from a north or south line, or meridian, agreed upon as a starting place, is called **longitude**.

It has been agreed that the starting place for measuring longitude shall be the Meridian of Greenwich, England, where there is an astronomical observatory. The distance halfway around the world to the west from Greenwich is called west longitude, and halfway around the world to the east from Greenwich, east longitude.

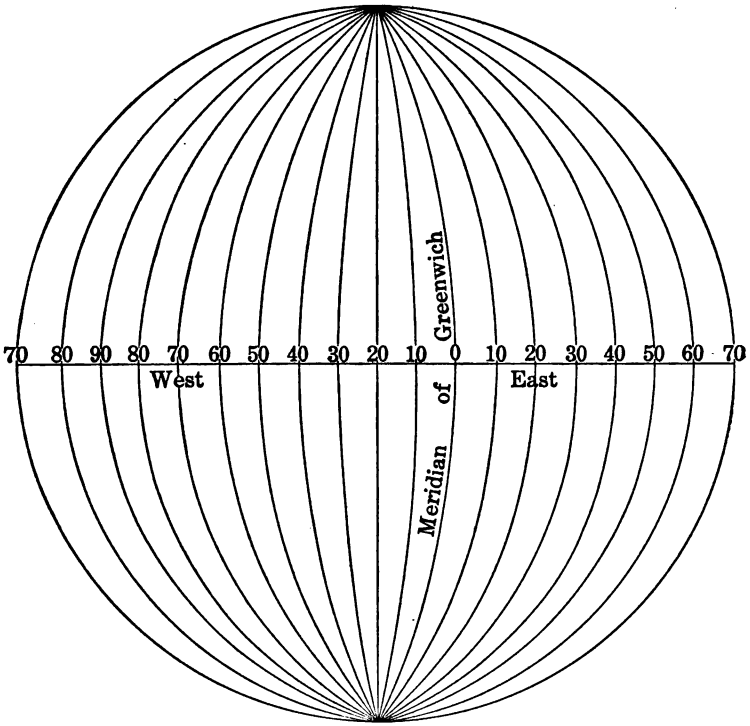
Is your meridian in east or west longitude?

142. Every circle, large or small, is supposed to be divided into 360 parts called degrees, by which it is measured. Longitude is measured in degrees. Being a circle, the distance around the earth is divided into 360 degrees (360°); 180° east longitude and 180° west longitude. Each degree is divided into 60 minutes (60′), and each minute into 60 seconds (60″).

60 seconds (60″) make 1 minute.
60 minutes (60′) make 1 degree.
60 degrees (60°) make 1 circle.

TIME

143. Any meridian may be considered as 0, but the one most commonly used is the Meridian of Greenwich, which is shown on the following cut:



Time is determined by the position of the sun. When the sun is directly overhead, it is noon. As the sun appears to move from east to west, it is noon at all places on any north and south line, or meridian, at the same time.

When it is noon where you are, is it past noon or before noon at places east of you? Places west of you?

The difference in time between two places is 4 hr. 10 min. 30 sec. When it is 20 minutes past 5 in the easterly place, what time is it in the westerly place?

5 hr.	20 min.	00 sec.
4 hr.	10 min.	30 sec.
1 hr.	9 min.	30 sec.

The answer is 9 min. 30 sec. past 1.

EXPLANATION: As we wish to find the time in the westerly place, we subtract the difference in time from the easterly time. Explain why.

The difference in time between two places is 3 hr. 50 min. 10 sec. When it is 20 minutes past 9 in the westerly place, what time is it in the easterly place?

9 hr.	20 min.	00 sec.
3 hr.	50 min.	10 sec.
13 hr.	10 min.	10 sec.

EXPLANATION: As we wish to find the time in the easterly place, we add the difference in time to the westerly time. Tell why.

The answer is 10 min. and 10 sec. past 13 o'clock; or, 10 min. and 10 sec. past 1 P.M.

Why is 13 o'clock the same as 1 o'clock in the afternoon?

To find the time at a given place from an easterly place subtract the difference in time; or, to a westerly place add the difference in time.

It is 20 minutes after 8 P.M. in a certain place when it is 30 minutes to twelve in another. What is the difference in time? In what direction is the first place from the second?

11 hr.	30 min.
8 hr.	20 min.
3 hr.	10 min.

We find the difference of time by subtraction. The place of later time will be east of the other place.

A place called A is 3 hr. 40 min. east of a given place, while a place called B is 4 hr. 50 min. west. What is the difference in time between A and B?

3 hr.	40 min.	East
4 hr.	50 min.	West
8 hr.	30 min.	

Inasmuch as one place is east and the other is west, the difference in time is found by adding.

LONGITUDE AND TIME

144. The sun appears to move around the earth in 24 hours. Hence, it appears to move through 360° in 24 hours, or 1440 minutes. To move through one degree it will take $\frac{1}{360}$ of 1440 minutes, or 4 minutes. Hence, the difference in time between two places 1 degree of longitude apart is 4 minutes.

The difference in time between two places is 6 hr. 10 min. What is the difference in longitude?

SOLUTION: 6 hr. 10 min. = 370 min.

$$4\overline{)370 \text{ min.}}\qquad 92° 30'$$

EXPLANATION. Since the sun passes through 1 degree in 4 minutes, it will pass through as many degrees in 6 hr. 10 min., or 370 minutes, as 4 minutes are contained in 370 minutes = 92 times and 2 remainder = $92\frac{2}{4}°$.

$\frac{2}{4}$ of 60 minutes = 30 minutes The answer is 92° 30′.

To find the difference in longitude when the difference in time is given, divide the number of minutes by 4.

The difference in longitude between two places is 15° 12′ 15″. What is the difference in time?

SOLUTION:

$$\begin{array}{r} 15°\ \ 12'\ \ 15'' \\ \hline 60'\ \ 49''\ \ 0 \end{array} \quad \begin{array}{l} \\ 4 \end{array}$$

or 1 hr. 49 sec.

EXPLANATION: The earth in its rotation passes over 1° of longitude in 4 min. of time, 1′ of longitude in 4 sec. of time.

Therefore, the number of degrees difference in longitude between two places × 4 = the number of minutes difference in time.

The number of minutes difference of longitude × 4 = seconds of difference in time.

(1) A is 40° 45′ 35″ east longitude and B is 65° 30′ 40″ east. The difference in longitude is found by subtracting the less from the greater.

(2) C is 15° 20′ 40″ west longitude and D is 42° 16′ 18″ west. The difference in longitude is found by subtracting the less from the greater.

(3) E is 24° 20′ 45″ east and F is 30° 15′ 30″ west. The difference in longitude is found by addition. Why?

From your geographies find the longitudes of Chicago, New Orleans, St. Louis, St. Paul, San Francisco, New York, Philadelphia, Boston, and of your own city or town.

1. Find the difference in longitude between your town or city and each of the above-named cities.

2. Find the longitude of Yokohama, Peking, Calcutta.

3. Find the difference in longitude between your town and each of the cities named in Problem 2.

4. To find the difference between a place in east longitude and one in west longitude do you add or subtract? Explain.

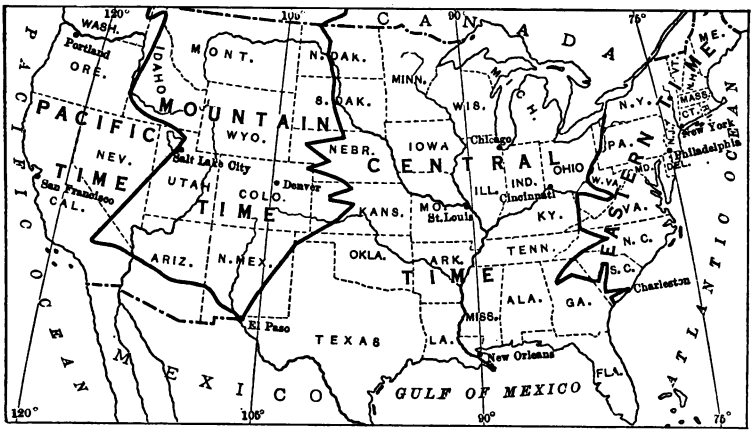
5. When it is noon in London, what time is it in New York? Do you add or subtract to find it?

6. In taking a long journey, I changed my watch every 15° of longitude. How much must I have changed it each time? I was traveling westward. In which direction did I change it?

7. If you were to travel exactly halfway around the earth without changing your watch, would it then need changing? Explain why?

TIME BELTS

145. Each meridian has its own time. No two places have the same time unless they are on the same north and south line, or meridian. As it would be very inconvenient to have our watches wrong whenever we go to a place only a short distance east or west, the governments of this continent have agreed to change time only when the difference is a whole hour, or until the earth has revolved through 15°. This is called Standard Time.



There are four divisions, or "time belts," on this continent: Eastern, Central, Mountain, and Western.

The **eastern time** is that of the 75th meridian, the **western,** that of the 120th.

1. How many hours difference (Standard Time) between New York and San Francisco?

2. In traveling across the continent from San Francisco to New York, how many times would you have to set your watch to keep Standard Time?

3. In going west, would you move your watch backward or forward? In going east?

4. How many times was it necessary for the men in the New York to Paris motor car race, by way of San Francisco, to change their watches in their race around the world?

5. How many times was it necessary to change their watches in crossing the United States? Asia? Europe?

6. Were their watches set back or ahead in each case? Explain why.

REVIEW

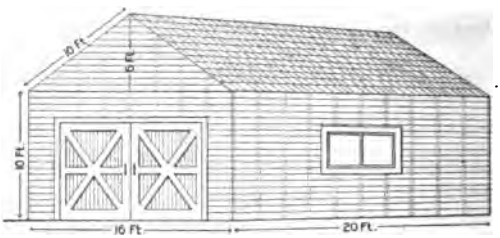
146. Solve:

(See Tables, p. 264.)

1. How many loads* in a bank of sand 6 ft. high, 20 ft. long, and 15 ft. wide?

2. How many loads in the excavation of a cellar, 25 ft. long, 16 ft. wide, and 8 ft. deep?

3. A contractor received 75 cents a load for excavating a cut for a road, 116 ft. long, 20 ft. wide, and averaging 15 ft. deep. How much did he receive?



This picture is a representation of a garage. The frame is constructed of timber, 3 in. by 4 in., each stick used being 10 ft. or 16 ft. long and used with the least waste. The rafters are 2 ft. apart. The joists, represented by the dotted lines, are also 2 ft. apart all around the main part of the building.

4. How many sticks of timber, 16 ft. long, must be used in building this garage? How many sticks of timber, 10 ft. long, can be used?

5. Allowing three 16-ft. sticks of timber for the gable ends, how many feet of timber were used? What is it worth at $3\frac{1}{2}$ cts. per running foot?

* In measuring excavations the unit is 1 cubic yard, which is called a *load*.

6. Making no allowance for windows, doors, or waste, how many square feet of boards would it take to cover the main part of the building? How many square feet in the gable?

7. How many square feet in the roof? What would the boards to cover the sides and ends of the building cost at $40 per M?

8. (Shingles are laid 4 in. to the weather and a shingle is 4 in. wide. Each shingle covers 16 sq. in., or there are 9 shingles to a square foot.) How many shingles did it take to shingle the roof? What would they cost at $8 per M?

9. Allowing $7.50 for extra work on the doors, and $5 for the windows, and 12 days' carpenter work, at $4.80 a day, what was the cost of the building?

10. The survey for a new road made the length 312 chains, 75 links. How long was it in miles, rods, and feet? In miles and decimals of a mile?

11. It is 12 miles and 3 rods from the post office of Amity to that of Coleville. How many chains and links?

12. The steamship *Adriatic* draws 22 ft. of water. How many fathoms deep must the water be for her to sail in?

13. How many ℈ in a pound? In an ℥?

14. How many ʒ in 1½ lb.?

15. Write as decimals of a pound, 3 ℥ 4 ʒ 50 ℈.

16. A gold dollar weighs 25.8 grains. What is the weight of a gold eagle? Of $1,000,000 of gold?

17. A silver dollar weighs 412.5 grains. What is the weight of $1,000 in silver?

18. If you are worth your weight in silver, how many dollars are you worth? (Reduce to grains. Why?)

19. The longitude of Berlin, Germany, is 13° 23′ 43.5″ E. That of Dublin, Ireland, is 6° 20′ 30″ W. When it is 7 A.M. at Dublin, what time is it at Berlin?

20. A man bought a plot of land in the form of a trapezoid. The length of one side was 256 ft., and that of the other side parallel to it was 328 ft. The distance between the parallel sides was 110 ft. How many acres did the plot contain?

21. Find the difference in area between a rectangle 36 ft. long and 24 ft. wide, and a trapezoid whose parallel sides measure 30 ft. and 42 ft. respectively, the distance between the parallel sides being 24 ft.

22. Classify the following units of measurement, making 3 groups of them. In Group One place all that refer to linear measurement, in Group Two all that refer to area, and in Group Three all that refer to volume.

Acre, inch, square foot, cord foot, mile, yard, section, cubic yard, rod, cubic foot, perch, square, square inch, square mile, square yard, link, chain.

23. Draw a rectangle; a square; a circle; a right triangle; a trapezoid.

24. There are two fields, each of which has a perimeter of 180 rd. One of them is a square, and the other a rectangle 8 times as long as it is wide. Which is the larger field?

25. What is the length of the rectangular field?

SUMMARY OF CHAPTER III

DENOMINATE NUMBERS
- Reduction.
- The Fundamental Processes.

APPLICATIONS OF DENOMINATE NUMBERS.
- Measurements.
- Longitude and Time.

CHAPTER IV

PERCENTAGE — APPLICATIONS OF PERCENTAGE

FINDING PERCENTAGE

147. 1 Per cent means 1 in each hundred, or $\frac{1}{100}$. It is written 1 % or .01. **Per cent** (centum) means **by the hundred.**

Of what per cent is each of the following common fractions the equivalent:

$$\frac{1}{2}, \frac{1}{4}, \frac{1}{3}, \frac{1}{6}, \frac{1}{5}, \frac{1}{8}, \frac{1}{10}, \frac{1}{12}, \frac{1}{16}?$$

Write each as a decimal.

In decimals, one tenth is the unit of calculation; in percentage one hundredth is the unit, but the decimal notation is used.

The terms of percentage are base, rate, percentage, amount, and difference.

The **base** is the quantity on which the percentage is reckoned.

The **rate** is the stated number of hundredths of the base. It is read "per cent."

The **percentage** is the quantity that equals the stated number of hundredths of the base. It is the product of the base by the rate.

The **amount** is the sum of the base and the percentage.

The **difference** is the base less the percentage.

The **amount** is represented by 100 % + the rate.

The **difference** is represented by 100 % − the rate.

In the following example name each term:

$$\begin{array}{r} \$455 \\ .05 \\ \hline \$22.75 \\ 455 \\ \hline \$477.75 \end{array}$$

148. As the relations of the quantities expressed by these terms are always the same, a clear understanding of these relations will make all problems in percentage simple of solution.

The relations may be stated in terms either of an equation or of a proportion.

I. Equation:

Base × Rate = Percentage.
Percentage ÷ Base = Rate.
Percentage ÷ Rate = Base.
Base + Percentage = Amount.
Base − Percentage = Difference.

These formulas should be memorized.

II. Proportion:

Three terms are given and x is used to represent the fourth or missing term. Remember that 100 % always equals the whole of any quantity.

Find 6 per cent of $600.

SOLUTION:

By the equation: $\$600 \times .06 = \$36.$

By proportion: $6\,\% : 100\,\% :: x : \$600.$

$$x = \frac{\$600 \times 6}{100} = \$36.$$

NOTE. In finding the percentage the use of the equation is much simpler. This is true also of the greater number of problems for finding other terms. But in some the proportion may be used to advantage.

149. TO FIND THE PERCENTAGE.

What is 12 % of $400?

SOLUTION: $400 × .12 = $48.

To find the percentage, base and rate being given, multiply the base by the rate written as a decimal.

ORAL EXERCISES

150. Find the percentage:

BASE	RATE	BASE	RATE
1,200 ft.	7 %	4,000 yd.	4 %
32	8 %	300	50 %
$18.75	10 %	$1,500	33⅓ %
6	60 %	400	25 %

WRITTEN EXERCISES

151. Solve:

1. Mr. Labon, in dividing his estate of 1,600 acres, gave 15 % of it to his son, and 20 % of the remainder to his daughter. How many acres did his son receive? His daughter?

2. Mr. Murray bought 400 yards of silk. He sold to one customer 8.5 %, and to another 16¼ %. How many yards did he sell to each?

3. Mr. Jeffries built a house at a cost of $6,500. He sold it at a profit of 10⅕ %. How much did he make? For how much did he sell it?

4. He built another house at a cost of $7,000 and sold it at a loss of 6¼ %. How much did he lose?

5. Mr. Adams had a field 42 rods long and 25 rods wide. He sold 33⅓ % of it. How many acres did he sell?

6. Mr. Dunn bought 820 yards of carpet at $.96 per yard. He sold it at a profit of 18⅓ %. How much did he make?

7. Mr. Holden has 27,000 feet of white pine boards, and 54,000 feet of oak boards. The pine cost him $48.50 per thousand feet. The oak cost him $65.50 per thousand. If he sells the boards at a profit of 15%, how much will he make?

8. A merchant bought 18 yards of cloth at $1.75 a yard. He sold 12 yards at a gain of $33\frac{1}{3}\%$ and 6 yards at a gain of 25%. What was his entire gain? What was his selling price?

FINDING AMOUNT OR DIFFERENCE

152. TO FIND THE AMOUNT OR DIFFERENCE.

By the equation:

Amount = Base + Percentage.

Mr. Sturgis had 48 cows. He increased his stock 25 per cent. How many cows did he then have?

SOLUTION: $48 \times .25 = 12$;

$48 + 12 = 60$;

or, $48 = 100\%$;

$100\% + 25\% = 125\%$,

$48 \times 1.25 = 60$.

Difference = Base − Percentage.

Mr. Reid bought a motor car for $1,800 and after using it a year sold it for $33\frac{1}{3}\%$ less. What did he receive for it?

SOLUTION: $\$1{,}800 \times .33\frac{1}{3} = \$600$;

$\$1{,}800 - \$600 = \$1{,}200$ difference;

or, $100\% - 33\frac{1}{3}\% = 66\frac{2}{3}\%$;

$\$1{,}800 \times 66\frac{2}{3} = \$1{,}200$ difference.

To find the amount or difference, base and rate being given, find the percentage and add it to or subtract it from the base; or,

Add the rate to or subtract it from 100% and multiply the base by the sum or difference.

WRITTEN EXERCISES

153. Solve:

1. Mr. Collinson, a dealer in heavy wire fence, bought fence at the following rates:

Fence 3 ft. high . . .	$.40 per ft.
3 ft. 4 in. high . .	.44 per ft.
4 ft. high	.48 per ft.

He sold to one man 120 ft. of the $.40 fence, to another 80 ft. of the $.44 fence, and to the third 75 ft. of the $.48 fence. He made a profit of 20 %. How much did he receive for the three sales?

2. Gates cost Mr. Collinson as follows:

Single Gates		Double Gates	
3 ft. wide	$4.00	8 ft. wide	$8.50
4 ft. wide	4.40	9 ft. wide	9.75
5 ft. wide	5.00	10 ft. wide	10.00

Mr. Williams bought a single gate 4 ft. wide and a double gate 9 ft. wide, Mr. Gillen bought a single gate 5 ft. wide and a double gate 8 ft. wide, and Mr. Dalton bought 3 single gates each 5 ft. wide. If Mr. Collinson's selling price is an advance of 20 % on the cost, what will be the price of each one of the gates? What will he receive for the gates sold to the persons named?

3. Mr. Goldsmith has watches that cost him $12 and $25.50. He sold a $12 watch at a profit of 22 %, and a $25.50 watch at a profit of $18\frac{1}{5}$ %. How much did he receive for each watch?

4. Mr. Morton has several kinds of tea that cost him $.30, $.45, and $.65 per pound. Retiring from business, he sold the $.30 tea $12\frac{1}{2}$ % less than cost, the $.45 tea $16\frac{2}{3}$ % less, the $.65 tea 20 % less. What did he receive per pound for each kind?

5. In one year there were 6,578 deaths from tuberculosis in a certain state. By separating the sick from the well, the annual deaths from this disease were reduced in 5 years 46 %. How many died in the fifth year?

6. In a school having an average attendance of 1,278 pupils, the per cent of tardiness was $3\frac{1}{2}$. What was the average number of pupils present on time? The average number late?

7. There are 30 breeds of sheep known. $\frac{1}{10}$ of these are found in Asia, $\frac{1}{3}$ in Africa, $\frac{2}{15}$ in Europe, and the others in America. What per cent of the known breeds are American? How many of the known breeds are American?

8. 100 Angora goats will keep 40 acres of land cleared of brush. An Iowa farmer who owns 640 acres has a sufficient number of goats to keep 25 % of it clear. How many goats has he? How many goats would be needed to keep his land clear?

9. A goat will produce at a shearing 4 lb. of fleece. If 70 % of it is worth 30 ct. a pound, what is the value of the 30-cent fleece from the goats kept on the 640-acre farm?

10. If 30 % of the fleece is short hair (noils) worth 16 ct. a pound, what is the value of the entire product?

11. A dealer sold a cask of oil containing 32 gal. He agreed to deliver it for $5.12. In delivering it $6\frac{1}{4}$ % of the oil was spilled. How many gallons were spilled? How much less than $5.12 did the dealer receive?

12. Mr. Ewing had a library of 875 volumes. In a fire 44 % of them were destroyed. How many books had he left?

13. Mrs. Nelson's piano cost $842. She was forced to sell it for 33 % less than it cost. How much did she lose on it?

FINDING BASE

154. To find the Base.

Solution:

By analysis:

25 is 25 % of what number?

25 % = $\frac{1}{4}$.

If 25 is $\frac{1}{4}$, the whole number or

100 % = 4 × 25 = 100.

By the equation:

Base = *Percentage* ÷ *Rate.*

25 ÷ .25 = 100.

To find the base, rate and percentage being given, divide the percentage by the rate.

WRITTEN EXERCISES

155. Find the base:

Rate Per Cent	Percentage	Rate Per Cent	Percentage
1. $12\frac{1}{2}$ %	$22.50	**5.** 40 %	30 yd.
2. $16\frac{2}{3}$ %	$8.25	**6.** 25 %	$16.25
3. $8\frac{1}{3}$ %	20	**7.** $33\frac{1}{3}$ %	6.50
4. 10 %	45.5	**8.** $6\frac{1}{4}$ %	200 ft.

9. A milk dealer sold 10.5 quarts of milk, which was $16\frac{2}{3}$ % of all he had. How much milk had he? What per cent had he left?

10. Mary has $20, which is $8\frac{1}{3}$ % as much as Louise has. How much money has Louise? How much more has Louise than Mary?

Make five similar problems based upon the rates and percentages given above.

11. Miss Riley saves $4.50 each week that she teaches. She teaches 40 weeks a year and saves 18 % of her salary. What salary does she receive for the year?

12. 15 % of Mr. Rhodes' strawberry crop was 420 quarts. He averaged $6\frac{1}{2}$ cents a quart for his berries. What did he receive for his crop?

SOLUTION:

By the equation $420 \div .15 = 2800$;

$2800 \times \$.065 = \$182.$

By proportion: $100\% : 15\% = x \text{ qt.} : 420 \text{ qt.}$

$$x = \frac{420 \text{ qt.} \times 100}{15} = 2800 \text{ qt.}$$

13. Mr. Lathrop saves $8\frac{1}{2}$ % of his salary. What is his salary if he saves \$75?

14. During a violent storm 64 bushels of apples dropped from the trees in Mr. Hendrickson's orchard. This was $12\frac{1}{2}$ % of his entire crop. How many bushels had he in all?

15. Mr. James gave \$25,000 towards purchasing land for a public park, \$5,000 for grading, \$2,000 for a fountain, and \$6,400 for a bridge across the park. Mr. James' donation was 60 % of the entire cost of the park. How much was the cost?

16. The salt water which was obtained from the bottom of a mine of rock salt contained 8 % of its weight of pure salt. What weight of water was it necessary to evaporate in order to obtain 3,894 pounds of salt?

17. If the salt water contained 12 % of pure salt, how many pounds of water would it be necessary to evaporate to secure 11,568 pounds of pure salt? How many pounds if it contained .05? 6 %? 5 %?

18. In Los Angeles, there were at a certain date 10,032 homes owned by the occupants, which is 44 % of all the homes in the city. How many homes did the city contain?

19. In Philadelphia 196,248 homes were rented. This was 78 % of all the homes in the city. How many homes were there?

FINDING RATE

156. To find the Rate.

$8 is what per cent of $32?

Percentage ÷ Base = Rate.

Solution:

$$32)8.00(.25, \quad \text{or} \quad \frac{8}{32} = \frac{1}{4} = .25.$$

$$\begin{array}{r} 64 \\ \hline 160 \\ 160 \\ \hline \end{array}$$

To find the rate, base and percentage being given, divide the percentage by the base.

WRITTEN EXERCISES

157. Solve:

1. 200,000 immigrants settled in New York in one year. If the population of New York was 4,500,000 at the beginning of the year, what was the per cent of increase from immigration during that year?

2. In Chicago there are 258,582 homes rented and 86,435 homes owned by the occupants. What per cent of all the homes are owned by the occupants?

3. The population of the United States is about 80,000,000. That of the world is estimated at 16,000,000,000. What per cent of the people of the world live in the United States?

4. A graduate of a trade school received $3.50 a week at first. After 3 years she was receiving $10 a week. What was the per cent of increase?

5. According to a recent estimate there are in New York 12,000 persons employed as cigar makers. Of these 9,000 are men. What per cent are women?

6. There are employed as tailors in New York approximately 50,000 persons, of whom 8,000 are women. What per cent of the tailors are men?

REVIEW

158. Solve:

1. The area of a rectangle 3 inches by 4 inches is what per cent of the area of a rectangle 6 inches by 8 inches?

2. The area of a right triangle whose base is 4 inches and perpendicular 5 inches is what per cent of the area of a right triangle whose base is 8 inches and perpendicular 10 inches?

3. The circumference of a circle whose diameter is 14 feet is what per cent of the circumference of a circle whose diameter is 28 feet?

4. Land was sold for $8,000, which was a loss of 20%. What was the cost?

5. Land was sold for $12,000, which was a gain of 20%. What was the cost?

6. A house that cost $6,400 was sold at an advance of $37\frac{1}{2}\%$. What was received for the house?

7. A house that cost $5,400 was sold at a loss of $16\frac{2}{3}\%$. What was received for the house?

8. On goods that were sold for $600, 20% was made. If they had been sold for $550, what would have been the gain or loss per cent?

9. Goods damaged by fire were sold for $900, at a loss of 25%. What would have been the gain or loss had they been sold for $1,000?

10. Goods marked at $1,500 were sold for 20% less than the marked price. For what were they sold?

11. Goods marked at $800 were sold for 25% less than the marked price. For what were they sold?

12. A class was given 200 words to spell. Minnie misspelled 15, Clara misspelled 19, Richard misspelled 23. What per cent did each spell correctly?

13. What number increased by 25 % of itself equals 80?

14. What number decreased by 25 % of itself equals 120?

15. A woman spent 40 % of her money and had $80 left. How much money had she at first?

16. What number increased by $62\frac{1}{2}$ % of itself equals 5746.39?

17. What number decreased by $37\frac{1}{2}$ % of itself equals 875.15?

18. Mr. Dayton bought $4\frac{1}{2}$ tons of hay for $\$18\frac{1}{2}$ per ton, and sold it for $95.50. What per cent did he make, approximately?

19. A retail dealer bought 2 bunches of bananas, containing 16 dozen, for $1.20. If $\frac{1}{4}$ of them are spoiled, and he sells the remainder at 16 cts. a dozen, what per cent will he gain?

20. Mr. Martin bought a pile of wood 24 feet long, 6 feet high, and 4 feet wide for $25. He sold it for $6.50 per cord. What per cent did he make?

21. Mr. Dunlap sold Mr. Smith a horse for $322, which was 8 % less than its cost to him. Mr. Smith sold it for $375. What per cent more than the cost to Mr. Dunlap did Mr. Smith receive?

22. A house cost $3500. It rents for 18 % of its cost. At what price does it rent?

23. In a school of 1580 pupils, 45 % of them are boys. How many girls are there in the school? What per cent of the pupils are girls?

24. If a man works 8 hours a day and sleeps 8 hours, what per cent of the day has he for other purposes?

25. Four boys ran an 880-yard relay race, running equal distances. What per cent of a mile did they run?

26. What per cent of a mile did each boy run?

27. John, William, Henry, and David ran a 440-yard junior relay race. What per cent of a mile did they run? What per cent of a mile did each boy run?

28. In the season of 1905 the New York National Base Ball Team won the pennant, winning 105 games and losing 48 games. What per cent of the games did the team win?

29. In the season of 1906 the Chicago National Base Ball Team won the pennant, winning 116 games and losing 36 games. What per cent of the games did they win?

30. Mr. DeMott sold to the railroad enough ties to lay a mile of track, the ties being 2 feet apart and averaging 9 inches wide. He paid for them 20 cts. apiece standing, $5 a hundred for cutting, and $10 a hundred for carting. He sold them at a profit of 25 %. How much did he receive for them.

31. Mr. Rathburn built a house on a lot 35 feet wide, for which he paid $60 a front foot. The mason's contract was $2,650, the carpenter's 85 % more than the mason's, the plumber's 50 % of the mason's, and the painter's 35 % of the plumber's. He sold the house for 15 % more than the entire cost. How much did he receive for it?

32. The floor of a room 60 feet by 70 feet was laid with maple boards. 25 % was allowed for waste. The boards were bought for $42.50 per thousand feet, with 5 % off for cash. Two carpenters were 12 days laying the floor. They received $3.80 a day each. Allowing $3.75 for the nails used, what did the floor cost?

33. Two newsboys, Jim and Joe, invested $3.60 in three different evening papers, paying an equal amount for each. They bought one paper at the rate of three copies for 5 cts. and sold them for 2 cts. apiece; they bought a second paper at the rate of five for 3 cts.; and the third at the rate of five for 4 ct. The last two papers they sold for 1 ct. each. What per cent did they make on their investment?

34. Find the percentage of each of the seven players in the National Base Ball League having the highest averages for the year, 1906, as follows (add "assisted" to "put out"):

	Put Out	Assisted	Errors	Total Chances
First Baseman, McGann, New York . .	1391	83	8	1482
Second Baseman, Ritchie, Pittsburg . . .	326	439	27	792
Third Baseman, Arndt, St. Louis	108	139	9	256
Short Stop, Tucker, Chicago	288	472	45	805
Outfielder, Gessler, Chicago	27	4	0	31
Catcher, Bowerman, New York .	300	80	6	386
Pitcher, Raulbach, Chicago . . .	17	74	3	94

35. A house that cost $6,000 rents for $45 per month. The tax rate for 1906 was $1.25 per hundred dollars, the repairs cost $1\frac{1}{2}\%$, the insurance was $12, and the water tax $18. What per cent did the owner receive for the money invested in the house?

TRADE DISCOUNT

159. It is customary for wholesale merchants to issue catalogues containing lists of goods with prices. Frequently, for business reasons, they sell goods for less than the **list price**. To avoid changing the catalogue when prices are lowered, the merchants give their customers what is called **trade discount**; that is, a certain per cent of the list price is deducted or "taken off" the bill. For example, cloth listed at 60 cents a yard might be sold "10 off," which would mean that 10 % was to be taken from the list price of 60 cents.

What would be the selling price of goods listed at 60 cents per yard if sold 10 % off?

Sometimes there are two or more discounts. Then the price might be 60 cents, "5 and 10 off," which would mean that 5 % was to be taken from the 60 cents, and then 10 % of the remainder taken off.

What would be the real selling price in that case?

Is this more or less than "15 off"? How much?

WRITTEN EXERCISES

160. Solve:

1. Mr. Bannister, who has a wholesale hardware store, sold Mr. Atwood a bill of articles amounting to $750 and allowed him 20 % and 10 % off. What did Mr. Atwood pay for his articles?

2. Mr. Leland bought different articles from Mr. Atwood to the amount of $450. He was allowed 18 % and 12 % off. What did Mr. Leland pay?

3. Mr. Van Nest bought 25 doz. brass hooks at $2.50 a dozen. He was allowed 15 % and 5 % off. What did he pay for the hooks?

4. How much less would they have cost him if he had been allowed 20 % discount?

5. Mr. Bigelow bought a bill of goods amounting to $3,000. He was allowed 15 %, 10 %, and 5 % off. What did the goods cost him?

6. For how much less could he have bought them at a discount of 30 %?

7. The trade discounts on a piano listed at $800 were 20 %, 15 %, and 10 % off. It was sold at a discount of 45 %. For how much less than the trade discount was it sold?

8. A wholesale silk dealer offered goods listed at $4,000 for 20 % and 12½ % off for cash, or a discount of 30 % and a note for 30 da. without interest. Which was the better offer, money being worth 6 % interest?

9. Mr. Pierson bought an automobile listed at $4,000, for 20 %, 10 %, and 5 % off. What did he pay for it?

10. A plumber bought supplies amounting to $480, at 35 % discount and 5 % for cash. What was the net price?

11. A cotton manufacturer, on account of a fall in the market, marked his cloth down 10 % from the list price, and an additional discount of 5 % for cash. What per cent of the list price did he receive for cash sales?

12. What single discount is equivalent to two successive discounts of 20 % and 10 %?

13. What single discount is equivalent to two successive discounts of 10 % and 5 %?

14. A dealer in wood and willow ware buys his goods at an average of 10 % and 5½ % off from list prices, and sells them at list prices. What per cent does he make on his sales?

15. A hardware dealer offered a bill of goods to a customer for $1200 with a discount of ¼ off, or a discount of 15 %, 10 % and 5 %. The customer accepted the first proposition. Was it the better one?

COMMISSION

161. The percentage, or compensation, allowed an agent for transacting business is called a **commission.**

Mr. Sweet sold for Mr. Howard merchandise amounting to $1560 and received $2\frac{1}{2}$ % for transacting the business. What was his commission? What did Mr. Howard receive for his merchandise?

SOLUTION: $1560

.025

7800

3120

$39.000 = commission.

$1560 − $39 = $1521, Mr. Howard received.

ORAL EXERCISES

162. Give the commission on each of the following:

	AMOUNT OF SALES	RATE OF COMMISSION
1.	$ 800	$2\frac{1}{2}$ %
2.	1,200	$2\frac{1}{2}$ %
3.	1,600	$2\frac{1}{4}$ %
4.	9,000	5 %
5.	6,000	5 %
6.	4,000	$2\frac{1}{2}$ %
7.	2,000	$2\frac{1}{4}$ %

WRITTEN EXERCISES

163. Solve:

1. Of a tract of land containing 150 acres, 45 acres were sold at $75.; 24 acres at $96.50; 38 acres at $136; and the remainder at $150. The agent received 5 % on what was sold for less than $100 per acre, and $3\frac{1}{2}$ % on what was sold for more than $100 per acre. What was his commission? What did the owner receive?

2. Mr. Morris sent to his agent, A. J. Holland, 1,450 baskets of peaches, 425 barrels of potatoes, 364 barrels of apples, and 2,000 pounds of cabbage. The peaches were sold for $1.25 per basket, the potatoes for $2.75 a barrel, the apples for $3.45 per barrel, and the cabbage at 3¢ per pound. What was Mr. Holland's commission at 10 %?

3. If Mr. Morris paid for freight 15¢ on each basket of peaches, 25¢ on each barrel of potatoes and apples, and $\frac{3}{8}$¢ on each pound of cabbage, how much did he receive for his crop?

4. Jan. 1. Mr. Holbrook sent his agent $2,000 to purchase flour. The agent bought 235 barrels at $5.40. His commission was 5 %. What was the amount of the commission? What amount remained to the credit of Mr. Holbrook?

SOLUTION:

$5.40 × 235 = $1269	cost of the flour.
$1269 × .05 = 63.45	commission.
$3132.45	cost, including commission.
$2000 − $1332.45 = $667.55	amount to the credit of Mr. Holbrook.

5. Jan. 10. The agent purchased for Mr. Holbrook 80 tubs of butter, averaging 56 pounds a tub, for 28¢ a pound. What was his commission? How much money should Mr. Holbrook send his agent to pay him in full?

6. Jan. 20. Mr. Holbrook sent his agent $1,000 to purchase tea and coffee. The agent bought 25 chests of tea, averaging 37 pounds a chest, for 43¢ a pound, and 2,160 pounds of coffee at $16\frac{3}{8}$¢ a pound. What was his commission? What remained to the credit of Mr. Holbrook?

7. Jan. 31. The agent purchased for Mr. Holbrook 15 barrels of sugar, averaging 300 pounds a barrel, at $6\frac{1}{4}$¢ a pound. What was the commission? What amount should Mr. Holbrook send his agent to close the January account?

TAXES

TERMS DEFINED

164. During the year 1906, a certain large city of this country spent $4,182,456.85 in maintaining the city government, the public schools, streets, police force, and other institutions for the welfare of its citizens. The money all came from **taxes** paid to the city by people who lived in it or owned property there.

A small part of this tax was a **poll tax** (poll means head), a fixed amount paid by every man over twenty-one years of age. The balance was paid by owners of property at a fixed per cent of the value of the property.

Officials known as **assessors** placed a value upon all the property owned in the city. Then the amount of money required to pay the expenses of the government was estimated. From this the amount that each property owner was required to pay for every dollar of property owned by him was determined.

For convenience, property is divided into two classes, called **real** and **personal**. Real property or real estate consists of buildings and lands, property that is fixed in place. Personal property is movable property, as money, stocks, bonds, cattle, horses, and carriages.

Taxes levied as above stated are called **direct taxes**. Town, county, and state governments are supported by money received from direct taxes.

In addition to the above sources of income, many cities receive money from **licenses**. These licenses give permission to conduct certain kinds of business within the city, and are paid for at a fixed rate. Some of the kinds of business that pay licenses are peddlers, push carts, theaters, and liquor saloons. In this city money from licenses is not estimated.

In addition to the taxes for supporting the city government, a county tax is levied for paying the cost of the county government and other expenses, of which a city situated in the county must pay its share. Most of the courts of justice, as well as the office of the sheriff, belong to the county.

165. A **tax** is a sum of money assessed on persons or property for public purposes.

A **property tax** is a tax assessed upon property.

A **poll tax** is a sum of money levied on male citizens more than 21 years of age.

The **assessed value** of property is taken as a basis of taxation.

An **assessor** is the officer who appraises the property.

166. The assessments for the maintenance of the several departments of the city, for 1906, were as follows:

Assessing and Collecting Taxes	$55,000
Poor and Alms	49,000
Incidental Expenses for Public Buildings	25,000
Hospitals	136,000
Insurance	25,000
Interest on Temporary Loans	15,000
Public Schools	905,271.37
Streets and Sidewalks	390,000
Lighting Streets	320,000
Street and Water Department	49,000
Sewers	40,000
City Home	37,500
Fire Department	488,500
Public Library	54,423
Public Health Department	475,000
Police Department	520,000
Shade Trees	18,500
* Sinking Funds and Interest	536,450.91

* NOTE. A sinking fund is a fund set aside each year for the payment of certain of the city's debts that fall due in the future.

Sundry other expenses	$42,811.57
Total amount needed by the city for carrying on the government in all its branches, to be raised by taxation in 1906, was	$4,182,456.85
From this amount was deducted 54,000 polls, at $1	54,000
Leaving to be raised by property tax . . .	$4,128,456.85
The county tax was.	1,256,422.47
The total tax was	$5,384,879.32
The real estate of the city listed for taxation was .	239,801,926.00
The personal property	45,101,304.00
Total "ratables"	$284,903,230.00

The amount to be raised, $5,384,879.32 ÷ 284,903,230, the total ratables, = .0189, the tax rate on $1, which equals $1.89 on $100, or $18.90 on $1,000.

Tax rates are often stated as so many **points**, thus .0189 on $1 = $1.89 on $100 = 189 points.

ORAL EXERCISES

167. What is the rate on $1, on $100, and on $1000, from the following numbers of points:

185	190	115	214	226
201	222	216	122	135
126	302	120	167	250

WRITTEN EXERCISES

168. Solve:

1. What was the tax, for 1906, of Mr. Ballard, whose property was in the above-mentioned city, and was assessed for $47,500? He also paid a poll tax.

2. Mr. W's real estate was assessed for $250,800 and his personal property $75,000. What was his tax on each? What was his total tax?

3. Mr. Howard's store was assessed for $60,500 and its contents for $45,000. What was his tax?

4. The taxable property in a town is $596,400. The rate of taxation is $.015. What is the amount to be collected? How much is the collector's commission at $2\frac{1}{2}\%$?

NOTE. In some localities discounts are allowed for prompt payment of taxes, and penalties are exacted of delinquents, if taxes are not paid by a certain date, as from Dec. 21 at the rate of 12%.

The rate in another city was 2.26%.
The following discounts for the year 1905 were allowed:

From Oct. 20 to Oct. 31 inclusive . . .	1%
From Nov. 1 to Nov. 20 inclusive . . .	$\frac{1}{2}$ of 1%.
From Nov. 21 to Dec. 20 inclusive . . .	exact amount of tax.

5. Mr. Halsey's property was assessed for $47,500. He paid his tax Oct. 28. How much did he pay?

6. Mr Humphrey's real estate was assessed for $145,000, and his personal property for $65,000. He paid his tax Nov. 10. How much discount did he receive? How much tax did he pay?

7. Mr. Smith paid his tax Dec. 12, 1905. His property was assessed for $6800. How much was his tax?

8. Mr. Hammond's property is valued at $18,000 and is assessed for $66\frac{2}{3}\%$ of its face value. He paid his tax March 1, 1906. What was the amount of his tax?

9. The real estate in a certain county is assessed for $116,000,000; the personal property for $64,000,000; and there are $28,250 polls assessed at $1.50 each. The tax to be raised is $3,912,375. What is the rate?

10. What is Mr. Randall's tax, if his property is assessed for $36,875, and he is exempt from poll tax?

11. Mr. Howard's store is assessed for $60,500 and its contents for $85,260. He pays a poll tax. What is the amount of his tax?

12. Mr. Hill has three houses, assessed at $8,275, $14,207, and $9,840 respectively. What is his tax on the houses?

INDIRECT TAXES

169. Every year the United States government makes large appropriations of money to be expended for the support of the army, the navy, and for other purposes. This money is received from taxes, but these are of a different kind from the taxes paid for the maintenance of the city government.

City taxes are paid upon polls and upon the value of property located in the city. The taxes for the support of the United States government are paid upon the value of certain articles manufactured in the United States, and upon certain articles bought in other countries and brought here. They are called **indirect taxes.**

Indirect taxes are of two classes, **internal revenue** taxes and **duties.**

Internal revenue taxes are paid chiefly on the value of liquors and tobacco manufactured in the country.

Duties are taxes paid on the value of goods brought into the country.

Duties are paid upon certain goods only.

A list of the goods with the amount of tax each is to pay is called a **tariff.**

WRITTEN EXERCISES

170. Solve:

1. During the fiscal year ended June 30, 1905, the aggregate collections of the Internal Revenue Bureau were $234,187,976.37. It cost $4,705,296.32 to collect it. What per cent of the amount received was the cost of collection?

2. The aggregate collections of duties during the same period were $261,798,856.91, and the cost of collection was $9,115,499.91. What per cent of the amount received was the cost of collection?

INTERNAL REVENUE

171. A **proof gallon** of alcoholic liquors is a gallon containing half alcohol and half water. The distilled spirits of commerce are usually 90% proof.

WRITTEN EXERCISES

172. Solve :

1. The tax on proof spirits is $1.10 per gallon. What is the tax on distilled spirits of commerce per gallon?

2. Estimating 50 drinks per gallon, what is the approximate governmental tax per drink?

3. Approximately 18,630,100 drinks of distilled spirits are taken in the United States each day. At 2¢ a drink, what is the revenue for a day? For a year?

4. The tax on fermented liquors, as beer, wine, ale, etc., is $1 a barrel of $31\frac{1}{2}$ gallons. It is estimated that a barrel of beer yields 400 glasses. How much tax does the government receive from each glass of beer?

5. If the government receives from the tax on beer a daily revenue of $137,000, how many glasses of beer are drunk each day in the United States?

6. The tax on a cigar is $\frac{3}{10}$¢. If the smokers of the United States consume 19,158,333 cigars each day, what is the daily revenue of the government from this source?

7. The tax on cigarettes is $1.08 per thousand. If the average daily income from these is $9,180, what is the number of cigarettes consumed?

8. The tax on chewing and smoking tobacco is 6¢ per pound. During the fiscal year of 1905 the consumers of chewing and smoking tobacco paid a government tax at the rate of $54,984 per day. How many pounds of chewing and smoking tobacco were used each day in the United States?

DUTIES

173. Duties are of two kinds, **specific** and **ad valorem.**

A tax levied on goods without reference to their value is called a specific duty.

Specific duties are usually levied at a fixed sum on each pound, or yard, or other measure of quantity.

A tax levied upon goods at the cost at which they are invoiced in the country from which they were imported is called an ad valorem duty.

Sometimes both specific and ad valorem duties are levied on the same article.

WRITTEN EXERCISES

174. Solve:

1. What is the duty on 80 yards of silk invoiced at $2.25 per yard, duty 60%?

2. At what price per yard must the importer sell it to make $12\frac{1}{2}\%$?

3. What is the duty on 15 rolls of silk, 45 yards in a roll, invoiced at $1.75 per yard, duty 60%?

4. At what price per yard must the importer sell it to gain 20%?

5. The cost of a rug in Japan was $450. The duty was 40%. For how much must the importer sell it to make $16\frac{2}{3}\%$?

6. A merchant imported 850 yards of flannel invoiced at $62\frac{1}{2}¢$ per yard, duty 35%. What did it cost him?

7. At 20% what is the duty on 25 bales of wool, 420 pounds each, invoiced at $6\frac{3}{4}¢$ per pound?

8. A merchant imported 800 yards of velvet, invoiced at $87\frac{1}{2}¢$ per yard, duty $12\frac{1}{2}¢$ per yard. He sold it for 1.37\frac{1}{2}$ per yard. What per cent did he make?

9. If the specific duty on a dozen pairs of gloves is $2.50, how much is the duty on 720 pairs?

10. What is the duty on 40 pieces of carpet, 25 yards each, invoiced at $1.50 per yard, the specific duty being $12\frac{1}{2}$¢ per yard and the ad valorem duty 40%?

11. A merchant imported 70 rolls of woolen cloth, each containing 30 yards, invoiced at 95¢ per yard, duty 50%. He sold the cloth for $1.75 per yard. How much did he make?

12. An importer bought cut glass in Germany, invoiced at $45,000, when the duty was 35% ad valorem. He sold one-half of it at a profit of 25%. After his purchase the tariff was raised to 60%. He sold the other half at a profit of 20% on the invoice price, plus the new duty. How much more was his gain on the last half of his stock than on the first half?

13. A cigar dealer imported 950 pounds of tobacco, invoiced at $4.50 per pound. The ad valorem duty was 25% and the specific duty $2.75 a pound. How much was the duty?

14. A lady brought from Europe for her own use, 25 yards of lace worth $1.25 per yard, upon which the duty was 50% ad valorem; 6 pairs of kid gloves upon which the duty was $2.25 per dozen pairs; a dress pattern upon which the duty was $27.50; and 15 yards of linen at 80¢ per yard, ad valorem duty 45%. How much more would the goods have cost her had she paid the duty?

15. Mrs. Lane brought from Germany 23 yards of lace invoiced at $2.35 a yard, and 12 yards of cloth invoiced at $1.90 a yard. The duty on the lace was 70% and that on the cloth 50%. What was the duty on each? What was the entire cost of both?

NOTE. Persons are allowed to bring from abroad a limited amount of goods for their own use, without paying duty upon them.

INTEREST

TERMS DEFINED

175. Interest is a sum paid for the use of money. It is reckoned as a percentage of the amount loaned for a specified time.

The terms used are principal, rate, interest, time, and amount.

The **principal** is the base upon which the interest is reckoned.

The **rate** is the stated number of hundredths of the principal.

The **interest** is the percentage of the principal for the stated time at the given rate.

The **time** is the period for which interest is reckoned.

The **amount** is the sum of the principal and interest.

METHODS OF COMPUTING INTEREST

176. There are various methods of computing interest. Some are better for short time loans, others for long time loans. Banks commonly use tables in which the interest on certain amounts for different periods is stated (see p. 134).

177. The Direct or Aliquot Part Method

This is especially simple when the time includes a number of months that are simple fractions of 1 year.

Find the interest on \$186 for 3 yr. 3 mo. at 5 %.

Solution:

	\$186			
	.05			
	\$9.30	× 3 =	\$27.90	interest for 3 yr.
4)	\$9.30	=	2.325	interest for 3 mo.
			\$30.225	

WRITTEN EXERCISE

178. Find the interest on:

1. \$285.50 at 5% for 2 yr. 6 mo.
2. \$378 at $4\frac{1}{2}$% for 4 yr. 4 mo.
3. \$720.25 at 7% for 3 yr. 3 mo.
4. \$187 at $3\frac{1}{2}$% for 5 yr. 2 mo.
5. \$865.45 at $6\frac{1}{2}$% for 1 yr. 9 mo.
6. \$236 at 6% for 6 yr. 1 mo.
7. \$240 at 5% for 8 yr. 8 mo.
8. \$350 at 5% for 4 yr. 6 mo.
9. \$975 at 4% for 5 yr. 8 mo.

179. SIX PER CENT METHOD

This method is used largely in computing interest when the time includes time units that are not easily found fractions of larger units, as: 5 mo. 17 da.

Interest is computed upon a year of 12 months of 30 days each, using the following formula:

Interest on \$1 for yr. $= \$.06$
Interest on \$1 for 1 mo. $= \frac{1}{12}$ of \$ .06 $= \$.005$
Interest on \$1 for 1 da. $= \frac{1}{30}$ of \$.005 $= \$.000\frac{1}{6}$.

180. Solve:

1. Find the interest on \$250 for 1 yr. 5 mo. 25 da. at 6%.

WRITTEN EXERCISES

SUGGESTION

Interest on \$1 for 1 yr. $= \$.06$ — \$.06
Interest on \$1 for 1 mo. $= \frac{.06}{12} = \$.005$
Interest on \$1 for 5 mo. $= 5 \times \$.005 = \$.025$
Interest on \$1 for 25 da. $= 25 \times \$.000\frac{1}{6} = \$.004\frac{1}{6}$
$\$.089\frac{1}{6}$

$\$250 \times .089\frac{1}{6} = ?$

2. Mr. Brown borrowed $450 of Mr. Avery, June 15, 1909. He gave his note on demand with interest at 6 %. He paid the note Feb. 21, 1910. How much did he pay?

NOTE. Interest at 3 % = $\frac{1}{2}$ interest at 6 %.
Interest at 2 % = $\frac{1}{3}$ interest at 6 %.
Interest at 4 % = $\frac{2}{3}$ interest at 6 %.
Interest at 5 % = $\frac{5}{6}$ interest at 6 %.
Interest at 7 % = $\frac{7}{6}$ interest at 6 %.

3. Find the interest on:

PRINCIPAL	TIME	RATE
$1,725.00	2 yr. 7 mo. 16 da.	3 %
1,650.00	5 yr. 3 mo. 10 da.	2 %
763.00	1 yr. 10 mo. 11 da.	4 %
927.50	8 mo. 17 da.	5 %
362.	1 yr. 8 mo.	6 %
215.45	7 mo.	4 %

SIXTY DAY METHOD

181. This is frequently used by banks.

182. Find the interest on $875, at 6 %, for 85 da.

EXPLANATION. 60 da. = 2 mo. = $\frac{1}{6}$ yr.

At 6% for 1 yr., the interest on any principal for 60 da. = $\frac{1}{6}$ of 6% = 1% = $\frac{1}{100}$ of itself.

85 da. = 60 da. + 20 da. + 5 da. Since any principal, at 6% draws $\frac{1}{100}$ of itself, in 60 da., interest on $875 for 60 da. = $8.75.

Since 20 da. = $\frac{1}{3}$ of 60 da., we find $\frac{1}{3}$ of $8.75 = 2.916\dot{6}$+, interest for 20 da.

5 da. = $\frac{1}{4}$ of 20 da. We find $\frac{1}{4}$ of $2.916 = $.729, interest for 5 da.

$8.75 + $2.916 + $.729 = $12.395.

SOLUTION:

$875 = principal.
3)8.75 = interest for 60 da.
4)2.916 = interest for 20 da.
.729 = interest for 5 da.
$12.395 85 da.

ORAL EXERCISES

183. Find the interest at 6% on the following:

1. \$148 for 60 da.
2. \$52.50 for 60 da.
3. \$720 for 30 da.
4. \$640 for 15 da.
5. \$820 for 75 da.
6. \$480 for 90 da.
7. \$240 for 10 da.
8. \$240 for 70 da.
9. \$900 for 60 da.
10. \$900 for 20 da.
11. \$300 for 80 da.
12. \$900 for 50 da.

WRITTEN EXERCISES

184. Find the interest on the following:

1. \$270.60 for 75 da. at 6%.
2. \$280.40 for 45 da. at 6%.
3. \$250.60 for 80 da. at 6%.
4. \$175.75 for 95 da. at 6%.
5. \$860.50 for 100 da. at 6%.
6. \$420.40 for 75 da. at 7%.
7. \$560.80 for 45 da. at 5%.
8. \$360.20 for 80 da. at 4%.
9. \$180.16 for 95 da. at 5%.
10. \$300.40 for 100 da. at $3\frac{1}{2}$%.
11. \$459.30 for 75 da. at 4%.
12. \$656.90 for 80 da. at 6%.
13. \$200.60 for 90 da. at 5%.

14. A man borrowed \$375 on May 1, 1906. He repaid it in 90 days with interest at 5%. How much did he have to pay?

15. On Feb. 2, 1905, Mr. Floyd borrowed \$670. He repaid it May 3, 1905, with interest at 6%. What amount did he have to pay?

Interest by Cancellation

185. What is the interest on $340.25, at 4%, for 99 da.?

The interest on $340.25, at 4%, for 1 yr., equals $340.25 $\times \frac{4}{100}$. The interest for 99 da. equals $\frac{99}{360}$ of the interest for 1 yr. $340.25 $\times \frac{4}{100} \times \frac{99}{360}$ = Ans. All factors common to both numerator and denominator are cancelled.

Solution:

$$\cancel{\$340.25} \times \frac{\cancel{4}}{\cancel{100}} \times \frac{\cancel{99}}{\cancel{360}} = \$3.74275$$

(.34025 · 11 · 90 · 10)

186. Find by cancellation the interest on the following:

1. $840 at 5% for 75 da.
2. $360.50 at 4% for 80 da.
3. $240 at 5% for 90 da.
4. $200 at 3½% for 48 da.
5. $80.84 at 4% for 18 da.
6. $960.40 at 4½% for 52 da.
7. $176.58 at 3% for 66 da.
8. $300 at 5½% for 36 da.
9. $980 at 4% for 72 da.
10. $19.50 at 4½% for 24 da.

Review

187. Solve:

1. The motor cars in the New York-Paris race left New York on February 12, 1908. On March 12 the American car, in the lead, had reached Bitter Creek, Colorado, having traveled 2,321 miles. What was the average number of miles a day?

2. The trip to San Francisco occupied 42 days. The distance from New York by the route followed is 4,836 miles. What per cent of the distance did the American car travel in the first month? In an average day?

3. The length of the Dnieper River is 1,250 miles; the Danube is 40% longer. How long is the Danube?

4. The length of the Elbe is 800 miles; the Rhine is 12½% shorter. How long is the Rhine?

5. The length of the Orinoco is 1600 miles; the St. Lawrence is $31\frac{1}{4}\%$ longer. What is the length of the St. Lawrence?

6. A gas bill reads as follows:

STATE OF METERS	
May 18, 1907,	16,000
April 18, 1907,	15,600
Consumption	400 cu. ft., at \$1.10 per 1,000 ft.

Discount of 10 ct. per 1,000 cu. ft., if bill is paid 5 days after presentation. Bill presented June 3. Find the amount of the bill and the discount.

7. John Jacob Astor left a bequest of \$400,000 to found the Astor Library, which was opened in the winter of 1853. William B. Astor, his son, gave during his lifetime a sum equal to 75% of his father's bequest, and left a bequest equal to $62\frac{1}{2}\%$ of his father's bequest. How much did William B. Astor give to the library? What was the whole amount contributed by father and son?

8. John Jacob Astor, 2d, gave to the library during his lifetime \$250,000 and left a bequest equal to $37\frac{1}{2}\%$ of all previous contributions. How much did he give to the library?

9. William Astor gave a sum equal to $12\frac{1}{2}\%$ of the bequest of John Jacob Astor, 1st. How much did he give to the library? What was the total amount contributed?

10. The population of Newark, N.J., was 240,000 in 1890; in 1900 it was 275,000. What per cent was the increase in population?

11. A watch gained 3 min. 45 sec. from 9.30 A.M. to 3 o'clock P.M. What per cent of the time did it gain?

12. Mr. Rutan sold two watches for $31.50 each. On one of them he lost $12\frac{1}{2}$ %, and on the other he gained $12\frac{1}{2}$ %. Did he lose or gain on the transaction?

13. In the 4-mile cross country run at Columbia University each class was represented by four runners. The class whose runners had the lowest average score won. How much less was their time than that of each of the other classes? What per cent less?

Seniors	
B. Sanders	22 min. 45 sec.
F. W. Kennedy	23 min. 20 sec.
F. Kudlich	23 min. 25 sec.
M. Smith	24 min. 11 sec.
Juniors	
J. W. Wheeler	23 min. 26 sec.
A. P. Montgomery	23 min. 54 sec.
H. N. Snevly	23 min. 55 sec.
F. Ware	25 min. 15 sec.
Sophomores	
F. L. Hopkins	24 min. 44 sec.
G. Hoynes	24 min. 50 sec.
G. E. Crunsky	25 min. 05 sec.
F. S. Hetherington	23 min. 0 sec.
Freshmen	
A. H. Meyer	25 min. 0 sec.
R. H. Seymour	25 min. 16 sec.
A. G. Marsh	26 min. 17 sec.
L. B. West	26 min. $55\frac{1}{2}$ sec.

14. Mr Grant sold 2 acres of land for $720 each. On one of them he gained 20 % and on the other he lost 20 %. What did the land cost him?

15. Find the difference between $2\frac{1}{2}$ % of $460.50 and $3\frac{1}{4}$ % of $380.40.

16. Mr. Kendall pays $25 a year for $1,000 life insurance. What rate per cent does he pay?

17. A house valued at $7,200 is assessed for $5,400. What per cent of its value is the assessment? If the tax rate is $2.32 on $100, how much is the tax?

18. A farm valued at $16,000 is assessed for 60 % of its value. What is the tax on the farm at the rate of $2.14 for $1,000?

19. Bennie bought 60 newspapers at the rate of 3 for 5 cents and sold them for 3 cents apiece. How much did he make? What per cent?

20. If Lena had taken 20 % more stitches she would have taken 720 stitches. How many stitches did she take?

21. On 4 days of the week Thomas has spelling lessons of 20 words each. On Friday all the words of the week are reviewed. If he misses an average of 5 words on Fridays, what is the average per cent of words spelled correctly? Of words missed?

SUMMARY OF CHAPTER IV

PERCENTAGE.
- Definitions and applications.
- To find Percentage, Amount or Difference, Base, Rate.

APPLICATIONS OF PERCENTAGE.
- TRADE DISCOUNT.
- COMMISSION.
- TAXES.
  - Direct.
  - Indirect.
- INTEREST.
  - Definitions.
  - Most-used Methods.
  - Direct or Aliquot.
  - Sixty Day.
  - Six Per Cent.
  - Cancellation.

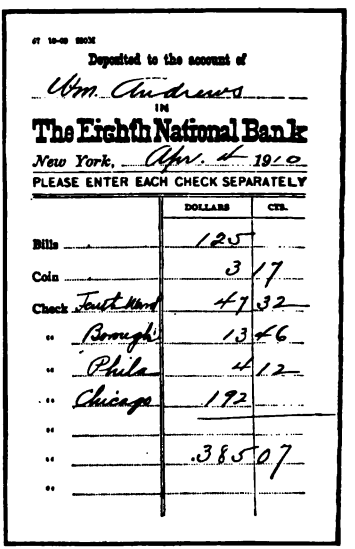
CHAPTER V

BANKING AND BUSINESS PRACTICE

METHODS OF DEPOSITING MONEY

188. Banks are institutions that deal in money and take care of other people's money. Most people do not keep much money on their persons or in their houses or places of business. They put it (or **deposit** it) in a **bank**. Money so intrusted to a bank is called a **deposit**.

Business men generally deposit every day the greater part of their cash funds. Commonly depositors use a slip like this, which the bank keeps to show just how much has been deposited.

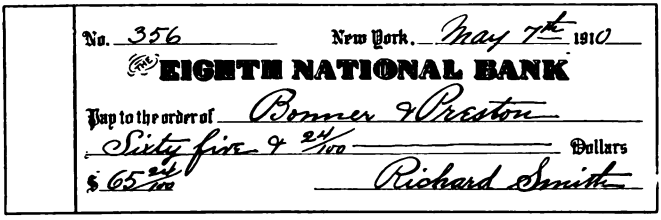

Deposited to the account of
Wm. Andrews
IN
The Eighth National Bank
New York, Apr. 4 1910
PLEASE ENTER EACH CHECK SEPARATELY

	DOLLARS	CTS.
Bills	125	
Coin	3	17
Check Fort Wayne	47	32
" Brooklyn	13	46
" Phila.	4	12
" Chicago	192	
"		
"	385	07
"		

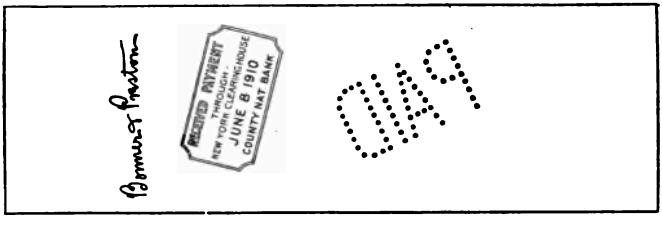
The bank clerk also enters the amount in a **bank book**, belonging to the **depositor**.

189. When depositors make payments to others, instead of paying out money they use checks. A **check** is an order on a bank to pay to some one an amount of money specified. The bank **honors** the check; that is, it pays the specified amount to the person named, after making sure that the one whose name is signed really signed it, and that he has money enough on deposit to pay it.

Read this check and tell on what bank the order was drawn. Who made the check? To whose order was it made? For how much was it made?



If the check is made payable to a particular person, he must **indorse** it, that is, write his name on the back, before he can get the money.



It is not necessary for the man who receives a check to go to the bank on which it is drawn to get his money.

190. Richard Smith owes Bonner & Preston $65.24. He has a deposit in the Eighth National Bank of New York. So he sends them a check for the amount of the bill. They indorse the check and deposit it, as if it were cash, in the Second National Bank, where they keep their money. The Second National Bank collects the amount, $65.24, from the Eighth National Bank, for them, and places it to their credit in that bank.

These banks thus save both of these parties a great

amount of time and trouble. By such means banks provide a convenient and prompt method of collecting and paying money. To pay themselves for this, they loan on interest the money deposited with them.

CLEARING HOUSES

191. For the convenience of banks in collecting from one another money due on the checks of their depositors, there are in most cities "**clearing houses.**"

A **Clearing House** is a place where representatives from the different banks meet to exchange checks and to adjust balances.

At a certain time every day the different banks send to the clearing house all the checks on other banks which have been deposited during the day. These checks are compared, the amounts due between the various banks canceled so far as they are equal, and any balance due paid in money.

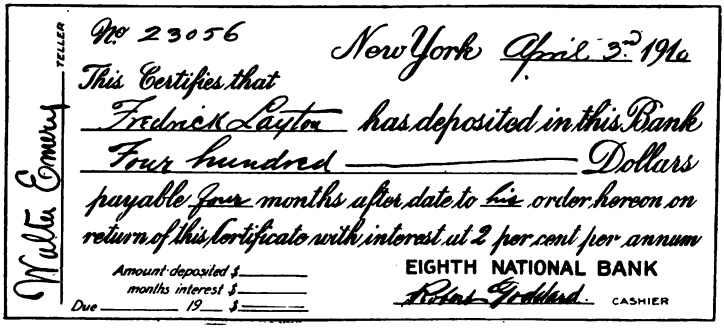
In the above example, the Second National Bank sends to the clearing house Mr. Smith's check drawn on the Eighth National Bank. The Eighth National Bank has a check drawn by Mr. Jones on the Second National Bank for $78.40. So the messenger of the Second National Bank pays to the messenger of the Eighth National Bank, in money, the difference between the two checks, $78.40 − $65.24 or $13.16. This is done through clearing house officials.

Each of the checks mentioned will be stamped on the back with the clearing house stamp which gives the date of its clearance.

192. All the checks due from each bank to all the others are taken by the officers of the clearing house, classified, and added. Then the **balances** are paid by those whose checks payable are less in amount than their checks receivable.

CERTIFICATE OF DEPOSIT

193. Banks receiving and caring for deposits do not pay interest on them, unless the depositors agree to leave the money in the bank for a considerable time. In this case the ordinary bank issues to the depositor a **certificate of deposit.**


No 23056 New York April 3rd 1916
This Certifies that
Frederick Layton has deposited in this Bank
Four hundred ——— Dollars
payable four months after date to his order hereon on
return of this certificate with interest at 2 per cent per annum
Amount deposited $———
months interest $———
Due ——— 19— $———
EIGHTH NATIONAL BANK
Robert Goddard CASHIER
Walter Emery TELLER

Sometimes interest is paid on the average of deposits left in the bank for 60 days or more.

WRITTEN EXERCISES

194. Find the interest on the average amounts of the following deposits for the periods named, at $3\frac{1}{2}\%$:

1. $135 from Feb. 2, 1907 to Feb. 10, 1908.
2. $1,275 " Mar. 6, 1908 " July 16, 1908.
3. $6,385 " Apr. 5, 1908 " May 7, 1909.
4. $10,150 " July 16, 1908 " Sept. 20, 1908.

5. Find the interest at 4% on the average amount of the following deposits for the periods named:

Sept. 1 to Nov. 15, $400.
Oct. 15 to Jan. 15, $375.
Nov. 1 to March 15, $1000.
Jan. 1 to March 15, $1200.
Feb. 1 to June 1, $1500.

TRUST COMPANIES AND SAVINGS BANKS

195. There are two kinds of banking institutions that receive deposits not subject to check and pay interest on them. Such banks are called **Trust Companies** and **Savings Banks.**

Trust companies usually receive money in large amounts, such as entire estates, and care for it, charging for time and services, and paying interest on funds deposited.

Some trust companies also do a regular banking business and pay out money on checks of depositors. But they do not pay interest on deposits subject to check, unless a special arrangement is made.

Savings banks receive money in small amounts and pay compound interest.

COMPOUND INTEREST

196. Savings banks and some other institutions pay interest on the money deposited with them. At stated times they add to the principal the interest then due, and compute interest on the amount for the next period, thereby paying not only interest on the original principal, but interest on interest.

This is called **compound interest.**

197. Interest is computed from a certain date in each month, usually not later than the 5th. Money deposited before that date receives interest from the 1st. Money deposited after that date draws no interest until the 1st of the next month. In some institutions no interest is allowed unless the money has been on deposit 6 months if compounded semiannually, or 3 months if compounded quarterly.

Money withdrawn from a savings bank before the end of any six months' period does not draw interest for that period if interest is compounded semiannually. Money withdrawn is called a **draft.**

198. Henrietta's father deposited in her name $600 in a savings bank. The bank pays its depositors 4 % interest, which is to be added to the principal semiannually. The money remained in the bank 3 years, when Henrietta withdrew it. How much money did she receive?

SOLUTION:	$600	First Principal.
	.02	Rate for one half year.
	$12.00	Interest for 6 months.
	600.00	
	$612.00	Second Principal.
	.02	
	$12.24	Interest for second 6 months.
	612.00	
	$624.24	Third Principal.
	.02	
	$12.4848	Interest for third 6 months.
	624.24	
	$636.725	Fourth Principal.
	.02	
	$12.7350	Interest for fourth 6 months.
	636.725	
	$649.460	Fifth Principal.
	.02	
	$12.9892	Interest for fifth 6 months.
	649.46	
	$662.449	Sixth Principal.
	.02	
	$13.2489	Interest for sixth 6 months.
	662.449	
	$675.697	Amount received.

Find the interest for the first interest period. Add the interest thus found to the principal, and find the interest on the amount for the second interest period, and continue in this manner until the money is withdrawn.

WRITTEN EXERCISES

199. Solve:

1. What would have been the simple interest on the foregoing deposit? How much more was the compound interest?

2. Mr. Hugo has 3 sons. On the day when Edward was just 19 years 6 months old, Rudolph 18 years old, and Victor 16 years old, the father deposited $4,000 to the credit of each son. Each was to receive his money when 21 years old. The bank in which it was deposited pays 4 % interest, adding interest semiannually. How much will each receive when he is 21 years old?

3. On his fifteenth birthday, Harold Chase deposited $450 in a savings bank, which paid annual compound interest at 5 %. How much was due him when he was 21 years old? What would have been the amount at simple interest?

200. Most banks find interest by using tables already computed, giving the amounts on $1, for different periods at different rates.

AMOUNT OF $1, AT COMPOUND INTEREST, COMPOUNDED ANNUALLY

YEARS	2%	2½%	3%	4%	5%
1	$1.02	$1.025	$1.03	$1.04	$1.05
2	1.0404	1.05063	1.0609	1.0816	1.1025
3	1.06121	1.07689	1.09273	1.12486	1.15763
4	1.08243	1.10381	1.12551	1.16986	1.21551
5	1.10408	1.13141	1.15927	1.21665	1.27628

Find by the table the amount of $320, at 4 %, for 3 yr.

The amount of $1, for 3 yr, at 4 %, is $1.12486.

$1.12486 × 320 = $359.955.

WRITTEN EXERCISES

201. Find, by using the above table, the amounts of the following principals, compounded annually:

1. $874, at 5%, for 2 yr.

2. $346, at 4%, for 3 yr.

3. Mr. Glenny loaned Mr. Faulkner $500, at 5%, for 3 yr. 6 mo., the interest to be compounded annually. What was due when the time expired?

4. What is the amount of $420, at 4%, for 4 yr. 4 mo., interest compounded annually?

5. What is the interest on $200, at 4%, for 3 yr., compounded semiannually? Compounded annually?

202. Make five problems and compute the interest, using the following table:

SAVINGS BANK TABLE SHOWING INTEREST ON DIFFERENT AMOUNTS, ESTIMATED BY MONTHS, AT 3½%

	1 Mo.	2 Mo.	3 Mo.	4 Mo.	5 Mo.	6 Mo.
$ 5	$.01	$.03	$.04	$.06	$.07	$.09
10	.03	.06	.09	.12	.15	.17
15	.04	.09	.13	.17	.22	.26
20	.06	.12	.17	.23	.29	.35
25	.07	.15	.22	.29	.36	.44
30	.09	.17	.26	.35	.44	.52
35	.10	.20	.31	.41	.51	.61
40	.12	.23	.35	.47	.58	.70
45	.13	.26	.39	.52	.66	.79
50	.15	.29	.44	.58	.73	.87
100	.29	.58	.87	1.17	1.46	1.75
150	.44	.87	1.31	1.75	2.19	2.62
200	.58	1.17	1.75	2.33	2.92	3.50
250	.73	1.46	2.19	2.92	3.65	4.37

203. Interest is computed and added to the principal, Nov. 1 and May 1, in the following savings bank accounts.

1907. JOHN SMITH

	DRAFTS	DEPOSITS	INTEREST
Nov. 3		$500	$8.75
Dec. 3		50	.73
Jan. 3		100	1.17
Feb. 3		75	.66
Mar. 3		25	.15
Apr. 3		50	.15
			$11.61

1907. WM. R. LEE

	DRAFTS	DEPOSITS	INTEREST
Nov. 1		$500	$8.75
Nov. 28		50	.36
Dec. 24	$25		
Feb. 8		100	.29
Feb. 15	50		
Mar. 3		700	1.75
Apr. 1	400		
Apr. 3		50	.15
			$11.30

In the first account Mr. Smith made no drafts on the bank. Each deposit was made during the specified time and drew interest from its date to May 1. The amount of interest, $11.61, is added to the sum of the deposits, $800, and the amount, $811.61, is the new principal from May 1. The same process is repeated for each succeeding six months. The rate is $3\frac{1}{2}\%$.

In the second account, Mr. Lee has made several drafts. In such an account, interest is computed on the smallest balance during the six months, which is $500. The second deposit, $50, was made November 28, therefore it would have drawn interest from December 1; but December 24, there is a draft of $25, which is deducted from the last deposit, $50 − $25 = $25, on which the interest is computed for five months. February 8, a deposit of $100 was made, and February 15 a draft was made for $50. $100 − $50 = $50, which draws interest from March 1, two months. On March 3 $700 was deposited and April 1, $400 was withdrawn, $700 − $400 = $300, which drew interest from March 1, two months. On April 3, $50 was deposited, which drew interest one month. The amount of deposits, plus the interest, $11.30, equals $1,411.30, from which $475, the amount of drafts, is deducted. $1,411.30 − $475 = $936.30, balance in bank May 1. Rate $3\frac{1}{2}\%$.

WRITTEN EXERCISES

204. Solve:

1. Mr. R. S. Dunlap's account with The Citizens' Savings Bank, from May 1 to Nov. 1, 1907, was as follows:

THE CITIZENS' SAVINGS BANK

1907. To R. S. DUNLAP, DR.

	DRAFTS	DEPOSITS	INTEREST
May 1		$800.00	
June 4		75.00	
July 3		200.00	
Aug. 1		25.00	
Oct. 4		50.00	

What was due Mr. Dunlap Nov. 1? Consult the table.

2. Mr. C. H. Childs' account with the same bank, for the same period, was as follows:

THE CITIZENS' SAVINGS BANK

1907. To C. H. Childs, Dr.

	Drafts	Deposits	Interest
May 1		$600.00	
May 20		150.00	
June 15	$50.00		
July 7		100.00	
July 20	75.00		
Sept. 3		500.00	
Oct. 1	30.00		
Oct. 2		80.00	

What amount has Mr. Childs in bank Nov. 1, 1907?

3. The State Savings Bank had in 1908 deposits amounting approximately to $16,500,000. It paid interest at the rate of $3\frac{1}{2}\%$ on an average amount of $14,500,000 for the first half year and $15,000,000 for the second half year. How much was paid out in interest?

4. The bank loaned an average amount of $7,000,000 at 5%, and of $7,000,000 at $4\frac{1}{2}\%$. What was the income from interest on loans? What was the excess of interest received over interest paid?

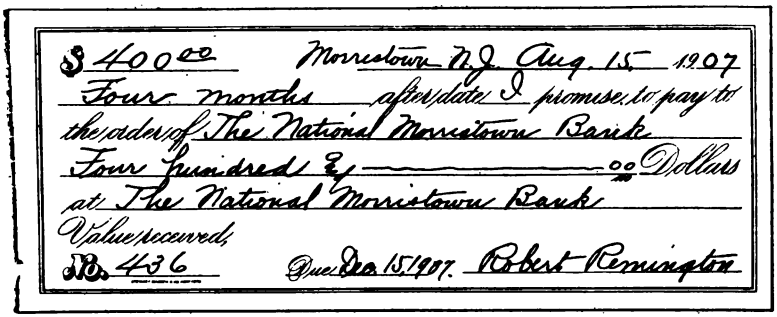
5. The expenses of the bank for rent, salaries, stationery, and other items, amounted to $35,500. What was the total surplus for the year?

6. The trustees, in view of the surplus, decided to raise the interest paid on deposits for 1909 one half of 1%. On the basis of the business for 1908, will this leave a surplus above all expenditures? How much?

LOANS

PROMISSORY NOTES

205. Individuals and banks loan money on interest, taking from the borrower a **promissory note** like this:


400^{00} Morristown N.J. Aug. 15 1907
Four months after date I promise to pay to
the order of The National Morristown Bank
Four hundred & ^{00}/$_{100}$ Dollars
at The National Morristown Bank
Value received,
No. 436 Due Dec 15 1907. Robert Remington

206. The borrower, who signs the note, is called the **maker.**

The note is the maker's promise to repay the amount borrowed. Sometimes the maker also promises to pay interest. Banks, however, usually require that the interest be paid in advance, or they deduct it from the amount loaned.

If a note is not paid when due, the holder may sue the maker; that is, ask the officers of the law to make the borrower pay.

207. Some notes specify a time of payment, as three months. They are called **time notes.** Notes given to banks for loans are almost always time notes and are seldom for a longer period than three or four months.

208. Some notes are payable on **demand**; that is, whenever the lender wants the money. They are called **demand notes,** and are usually expected to be paid soon.

209. Frequently banks require that the note shall be **indorsed** by some other person. **Indorsing** a note is writing one's name on the back of it. The person so indorsing becomes responsible for the payment of the note and may be compelled to pay it, if the maker fails to pay.

210. Usually the borrower puts in the bank **security**; that is, some property which the bank can sell in order to get its money, in case both the maker and the indorser fail to pay it when due.

Banks, other than savings banks and trust companies, usually require that the security given with notes be in the form of stocks, or bonds, or other property for which there is always a market, so that there may be no delay in collecting the debt, if the bank needs the money quickly to pay checks or drafts.

BANK DISCOUNT

211. Banks loaning money usually require the interest to be paid in advance. The interest is deducted from the loan.

This practice of requiring that the interest be paid in advance causes the borrower to pay interest on a greater amount than that specified. Although this makes a difference that, in ordinary transactions, is small, still in the course of a year the advantage to a bank is very great. This difference can be seen in the example given below.

On Aug. 15, Mr. Remington gives his four months' note to the bank for $400. But he receives $400 less the interest for 4 months. When the 4 months are past, he pays the bank exactly $400.

212. Interest paid in advance is called **Bank Discount.**

Since notes are seldom discounted for more than 4 months, the discount is most easily found by the 60-day method, computing the interest for the exact number of days between the dates.

$400 face of note.

SOLUTION: Days of discount in August, 16
" " " " September, 30
" " " " October, 31
" " " " November, 30
" " " " December, 15
Total days of discount 122

$8.00 = discount for 120 days at 6%.
.13 = " " 2 " " 6%.
$8.13 " " 122 " " 6%.

$400.00 − $8.13 = $391.87 = the amount received by Mr. Remington.

213. The difference between the face of the note and the bank discount is called the **proceeds.** In this example, $391.87 is the proceeds.

If Mr. Remington had paid interest on the amount actually received, how much would he have paid?

How much did the bank gain by this method?

To find bank discount, compute the interest on the face of the note from the date of discount to the date of maturity. The discount deducted from the face of the note gives the proceeds.

ORAL EXERCISES

214. Find the discount, at 6%, on the following:

1. 60-day note for $125.
2. 60-day note for $160.50.
3. 30-day note for $840.
4. 30-day note for $240.50.
5. 15-day note for $600.
6. 75-day note for $480.
7. 90-day note for $150.
8. 90-day note for $120.
9. 20-day note for $630.
10. 80-day note for $280.
11. 10-day note for $636.
12. 70-day note for $240.

215. Bankers in computing discount commonly use interest tables already computed.

100 Dollars

Days	½ per ct.	1 per ct.	2 per ct.	3 per ct.	3½ per ct.	4 per ct.	4½ per ct.	5 per ct.	6 per ct.
1	0	0	01	01	01	01	01	01	02
2	0	01	01	02	02	02	03	03	03
3	0	01	02	03	03	03	04	04	05
4	01	01	02	03	04	04	05	06	07
5	01	01	03	04	05	06	06	07	08
6	01	02	03	05	06	07	08	08	10
7	01	02	04	06	07	08	09	10	12
8	01	02	04	07	08	09	10	11	13
9	01	03	05	08	09	10	11	13	15
10	01	03	06	08	10	11	13	14	17
11	02	03	06	09	11	12	14	15	18
12	02	03	07	10	12	13	15	17	20
13	02	04	07	11	13	14	16	18	22
14	02	04	08	12	14	16	18	19	23
15	02	04	08	13	15	17	19	21	25
16	02	04	09	13	16	18	20	22	27
17	02	05	09	14	17	19	21	24	28
18	03	05	10	15	18	20	23	25	30
19	03	05	11	16	18	21	24	26	32
20	03	06	11	17	19	22	25	28	33
21	03	06	12	18	20	23	26	29	35
22	03	06	12	18	21	24	28	31	37
23	03	06	13	19	22	26	29	32	38
24	03	07	13	20	23	27	30	33	40
25	03	07	14	21	24	28	31	35	42
26	04	07	14	22	25	29	33	36	43
27	04	08	15	23	26	30	34	38	45
28	04	08	16	23	27	31	35	39	47
29	04	08	16	24	28	32	36	40	48
30	04	08	17	25	29	33	38	42	50

WRITTEN EXERCISES

216. Using the table, find the proceeds, at 6%, on:

1. A 15-day note for $180.50.
2. A 25-day note for $415.75.
3. A 30-day note for $270.50.
4. Find the proceeds of Example 1 at 7%.
5. Find the proceeds of Example 2 at 5%.
6. Find the proceeds of Example 3 at $4\frac{1}{2}$%.

Assuming that each of the following notes was discounted for the time given, find the discount on:

7. A 2 months' note, for $340, dated April 4, 1906, at 6%.
8. A 3 months' note, for $160.50, dated May 12, 1906, at 6%.
9. A 4 months' note, for $420.80, dated Oct. 5, 1905, at 6%.

In the following examples find

(1) Date of maturity.
(2) Time for which the note was discounted.
(3) Discount.
(4) Proceeds.

10. A note for $225.50, June 8, 1905, for 3 months, was discounted July 15, 1905. Rate, 5%.

Date of maturity, Sept. 8, 1905.
Time from July 15 to Sept. 8; time for which note was discounted, 55 days.
Discount, $1.71.
Proceeds, $223.78.

11. Face of note, $75.80; date, Jan. 16, 1906; time, 4 months; rate, 6%; discounted, March 12, 1906.

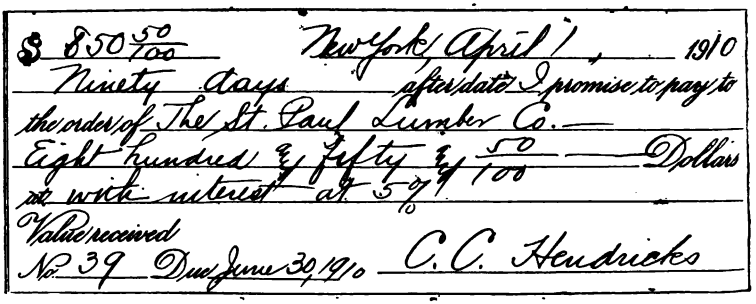
12. Date, Feb. 20, 1906; face of note, $320.60; time, 4 months; discounted, April 12, 1906; rate, 6%.

13. Date, March 11, 1906; face of note, $96.80; time, 90 days; discounted, May 14, 1906;. rate, 5%.

14. Date, April 1, 1906; face of note, $80.50; time, 100 days; discounted, June 24, 1906; rate, 6%.

INTEREST-BEARING NOTES

217. Most notes given for loans, whether *secured* by mortgages or not, except notes given to banks for short-time loans, require that interest be paid, not in advance, but when the principal is due or at stated times, as annually or semiannually. Such notes are called **interest-bearing notes.**


$ 850 50/100 New York, April 1, 1910
Ninety days after date I promise to pay to
the order of The St. Paul Lumber Co.
Eight hundred & fifty & 50/100 Dollars
at with interest at 5%
Value received
No. 39 Due June 30/10 C. C. Hendricks

218. Write two interest-bearing notes for different times, one at 4½% and one at 6%.

Compute the interest on each and give the amounts to be paid when the notes fall due.

219. Sometimes the holder of a note, instead of waiting until it is due, sells it to the maker or to some one else, and gets his money in advance. To do this, he must deduct the portion of the interest not yet due.

Accepting payment of a note less the interest to be earned is called **discounting** the note.

220. Sometimes the holder of a note bearing interest takes the note to a bank and has it discounted; that is, the bank pays him what the note is worth at the time. This is usually computed as the amount of the note less the interest for the time it has to run.

Mr. Gaynor gave Mr. Bannard his note for $600 for 6 mo. without interest. After 3 mo. Mr. Bannard sold the note to his bank and received its **present worth**. This was $600 less the interest for 3 mo. at the bank's regular rate of 5%. $600 − $7.50 = $592.50.

221. Sometimes a borrower, having paid the interest on a note in advance, desires to repay the loan before maturity. In such cases the bank deducts from the principal due the interest paid for the unexpired portion of time.

WRITTEN EXERCISES

222. Solve:

1. The Gem Manufacturing Company in payment for goods sold took the notes of their customers, without interest, as follows:

Jan. 1, 1909,	The Hackett Hardware Co. .	$575.50	3 mo.
Feb. 15, 1909,	The Cortlandt Company . .	$1587.75	4 mo.
Mar. 4, 1909,	H. S. Kendall	$5685.00	2½ mo.

On March 15 all the notes were sold to the Liberty Bank, which discounted them at 5%. What amount did the Gem Manufacturing Company receive?

SAN FRANCISCO, CAL., May 1, 1908.

2. Ninety days after date, for value received, I promise to pay G. C. Baker Six Hundred and Seventy-five Dollars ($675), with interest at 6%.

JOHN SCAIF.

How much should Mr. Baker receive?

On June 20 Mr. Scaif paid the note, with the interest then due. How much did he pay? How much interest did he save?

NEW YORK, April 3, 1908.

3. Sixty days after date, I promise to pay to the Liberty Bank, Three Hundred Dollars, with interest at 6 %. Value received.

HENRY SCHENCK.

How much did Mr. Schenck receive for the above note?

4. On May 3 he paid the note. For how much interest did he receive credit? How much did he have to pay to take up the note?

Make and solve five similar problems.

5. Write a note, correct in form and wording, from the following data:

Maker of note, yourself.
Payee, your teacher.
Face of the note, $1400.
Rate, the legal rate in your state.
Length of time, 1 yr. 3 mo. from date.
What will be the interest? The amount? What kind of a note is it?

223. Sometimes the holder of a note bearing interest, in order to secure payment before maturity, discounts it at a higher rate of interest than that named in the note. That is, he pays the difference between the discount and the interest so that he may get his money.

NEWARK, N.J., July 5, 1906.

Seventy-five days after date I promise to pay C. T. Shipman, or order, Ninety-six and $\frac{50}{100}$ Dollars, value received, with interest at 5 %.

F. N. DOREMUS.

The above note was discounted Aug. 21, 1906, at 6%. Find proceeds.

SOLUTION: \$96.50 face of note.
\$ .965 interest for 60 days.
\$ .241 interest for 15 days.
\$1.206 interest for 75 days at 6%.
\$1.206 ÷ 6 = \$ .201 interest for 75 days at 1%.
\$ .201 × 5 = \$1.005 interest for 75 days at 5%.

\$96.50 + \$1.005 = \$97.505, amount of note at maturity.
Date of maturity, September 18.
Time of discount, from Aug. 21 to Sept. 18, 28 days.
Discount on \$97.505, for 28 days, at 6% = \$.455.
\$97.505 − \$.455 = \$97.05, proceeds.

To find the proceeds, find the amount of the note for the given time and rate, and then find the discount on this amount from the date of discount to date of maturity. Deduct the discount from the amount of the note and the result will be the proceeds.

WRITTEN EXERCISES

224. Solve:

1. Mr. Delavan sold a house for \$8000. In payment he received a note dated Feb. 16, 1906, for 4 months, at 6%. He had the note discounted April 10, 1906, at 5%. How much did he receive?

2. Mr. Morris sold his farm for \$12,000, taking in payment a note dated January 12, 1906, for 3 months, at 5%. He had the note discounted 20 days before it was due, at 6%. What were the proceeds?

3. Find the proceeds of a 90-day note for \$180.36, bearing interest at 7%, discounted 36 days after date, at 6%.

4. Paul Brothers sold 25 tons of coal for \$6.50 a ton, taking in payment a note dated July 6, 1906, for 4 months, at 5%. They had the note discounted October 15, 1906, at 6%. What were the proceeds of the note?

MORTGAGES

225. Individuals who have money to loan, and also Savings Banks and Trust Companies, frequently take as security valuable property, like land or houses, that cannot be sold as quickly as stocks. As real estate cannot be deposited with a note, the security is put into a special form called a mortgage.

A **mortgage** is a contract giving the lender the right, in case a loan is not paid, to take possession of the property, have it sold by an officer of the law, and retain the amount due him.

WRITTEN EXERCISES

226. Solve:

1. Mr. Charles Gray bought a farm for $7000. He paid $2000 in cash and gave a mortgage for the balance. The mortgage was for 3 years with interest at 6%, payable semiannually. If the mortgage is paid when due, what will be the total cost of the farm including interest? Why did the seller take a mortgage besides the note?

2. Mr. Appleby, owning a farm, insured his barn and contents for $1,000. A fire destroyed it, including his stock, wagons, and hay. He built a new barn costing $1,200, bought cattle for $500, and wagons and implements costing $300.

To pay for these, besides his insurance money, he borrowed of the savings bank the amount needed, giving a mortgage on the farm at 5% for five years. What was the amount of the mortgage? How much interest did he pay yearly?

3. Each year he saved money to pay the mortgage when due. This money he deposited in the savings bank, where it drew interest at 4% compounded semiannually. If he deposited $300 each year, and paid the interest on the mortgage regularly, how much remained to his credit in the bank after paying off the mortgage at the end of five years?

EXCHANGE

227. Banks make it possible for persons living in different places to do business without sending sums of money from place to place.

The following will illustrate the method used:

Mr. Totten, in New York, buys oranges of the California Fruit Co. for $1,000. To send $1,000 in cash to California would be troublesome and expensive; therefore he sends his check for $1,000 on the Chemical Bank of New York to the company in California. This company indorses the check and deposits it in the First National Bank of Los Angeles.

The Angelus Hotel, of Los Angeles, has bought of McCutcheon & Co., of New York, $1,000 worth of linen. They send their check on the First National Bank of Los Angeles for $1,000. McCutcheon & Co. indorses the check and deposits it in the Chemical Bank.

The Chemical Bank of New York and the First National Bank of Los Angeles now each holds a check for $1,000 against the other. The two checks cancel each other, and the two debts of $1,000 each are paid without sending any money.

This is called **exchange.**

A plan similar to this is followed in sending money from one country to another. The banks act as agents and take charge of the transfer at very little cost to the senders. The liability of loss is small, because of the integrity of banks.

228. Exchange between places in the same country is called **domestic exchange.**

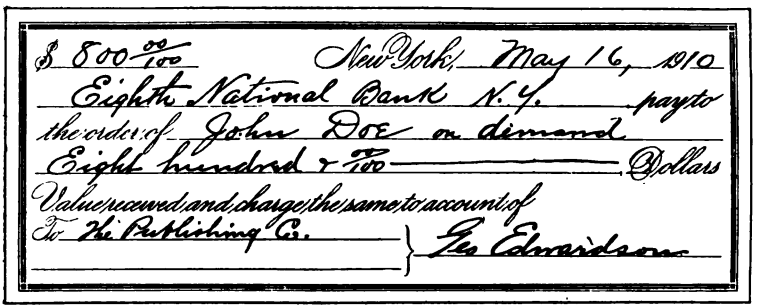
229. Exchange between places in different countries is called **foreign exchange.**

By means of exchange, very large business transactions can be carried on between distant places with the transportation of money enough to pay balances only.

FORMS OF EXCHANGE

230. By Bank Drafts.

For making payments at a distance there are other means besides checks. The most common are **bank drafts.**


$ 800 00/100 New York, May 16, 1910
Eighth National Bank N.Y. pay to
the order of John Doe on demand
Eight hundred & 00/100 Dollars
Value received and charge the same to account of
To the Publishing Co. Geo Edwardson

231. Mr. Bell, living in Winona, Minn., wants to send $320 to Mr. Hulbert, in Chicago. He goes to the First National Bank, of Winona, where he has a deposit, and asks for a draft on Chicago. The bank has an account with the First National Bank of Chicago, so Mr. Bell receives a draft. Mr. Hulbert either collects the money of the Chicago Bank, or deposits the draft in his own bank and the bank collects it for him, through the clearing house.

232. It is customary for banks to charge a small percentage to cover the various expenses of exchange. If payment is made by check, the bank finally receiving the check frequently deducts this cost from the amount paid.

Thus, if Mr. Bell had sent his check to Mr. Hulbert for $320, the Chicago Bank would probably have paid $320 − 15 cents, or $319.85. The amounts charged vary. As Mr. Bell sent a draft, he probably had to pay the Winona Bank 10 or 15 cents for it.

233. Foreign Exchange usually is charged at a fixed rate per cent.

234. For vegetables sold, a commission merchant sent returns which amounted to $2,400. If the exchange was $\frac{1}{8}$ %, what did the vegetables net the farmer?

235. By Postal Orders.

The government provides a convenient method of sending small sums of money through the post office, by means of **postal money orders.**

Following is a list of charges:

For orders for sums not exceeding							$ 2.50,	3	cents.
"	"	over	$ 2.50	and not	exceeding		5.00,	5	"
"	"	"	5.00	"	"	"	10.00,	8	"
"	"	"	10.00	"	"	"	20.00,	10	"
"	"	"	20.00	"	"	"	30.00,	12	"
"	"	"	30.00	"	"	"	40.00,	15	"
"	"	"	40.00	"	"	"	50.00,	18	"
"	"	"	50.00	"	"	"	60.00,	20	"
"	"	"	60.00	"	"	"	75.00,	25	"
"	"	"	75.00	"	"	"	100.00,	30	"

236. Make and solve five problems involving the sending of money by postal orders.

237. By Express Orders.

Express companies also sell money orders collectible at their own offices in different places. The rates are about the same as those of the post office.

238. By Telegraph Orders.

The telegraph companies furnish the quickest means of sending money. They sell orders and communicate with their agents by telegraph, so that a man in New York or London may send money to San Francisco to be paid the day

it is sent. The charges for this service are somewhat higher than for post office or express orders.

239. By Letters of Credit.

Persons traveling and not wanting to carry large sums of money may buy **letters of credit.**

240. Mr. Rising, of St. Paul, Minn., about to take a trip through Europe, put $5,000 in his bank in St. Paul. He received a letter of credit for $5,000, less 2%, the charge for exchange. This letter was addressed to banks in the cities of Europe that he planned to visit. At each of these banks he could draw money as he needed it, the amount drawn being entered on the letter. How much credit did he receive?

Make four similar problems.

241. English money is measured in pounds, shillings, pence and guineas.

12 pence (*d*) make 1 shilling.
20 shillings (*s*) make 1 pound (£).
21 shillings make 1 guinea.

The value of the pound as compared with the dollar varies according to demand, but generally £ 1 = $ 4.86 nearly.

242. The standard French coin is the franc, which is worth approximately 20 cents. The 5-franc piece corresponds nearly to one dollar.

243. The German standard coin is the mark, worth about about 24 cents; so that 4 marks equal approximately $1.

244. The Dutch standard is the guilder, worth about 40 cents.

245. In Austria the standard is the gulden, worth nearly 48 cents.

WRITTEN EXERCISES

246. Solve:

1. Mr. Rising collected in London $600 on his letter of credit. How much did he get in English money at $4.86. His hotel bill was £52. How many dollars were required to pay it?

2. He spent £3 for cab hire, 12*s.* for flowers, 10 guineas for clothes, and 5*s.* 6*d.* for tips. What did they amount to in dollars?

3. If a dollar was worth 5.2 francs, how many francs did Mr. Rising receive when he drew $750 in Paris?

4. Make and solve three problems covering his expenses in Paris.

5. Mr. Rising drew $800 in Berlin. With 4 marks worth 95 cents, how many marks did he receive?

6. Make two problems for his expenses in Berlin.

7. In Amsterdam he drew $650. With guilders worth 40.2 cents, how many did he get?

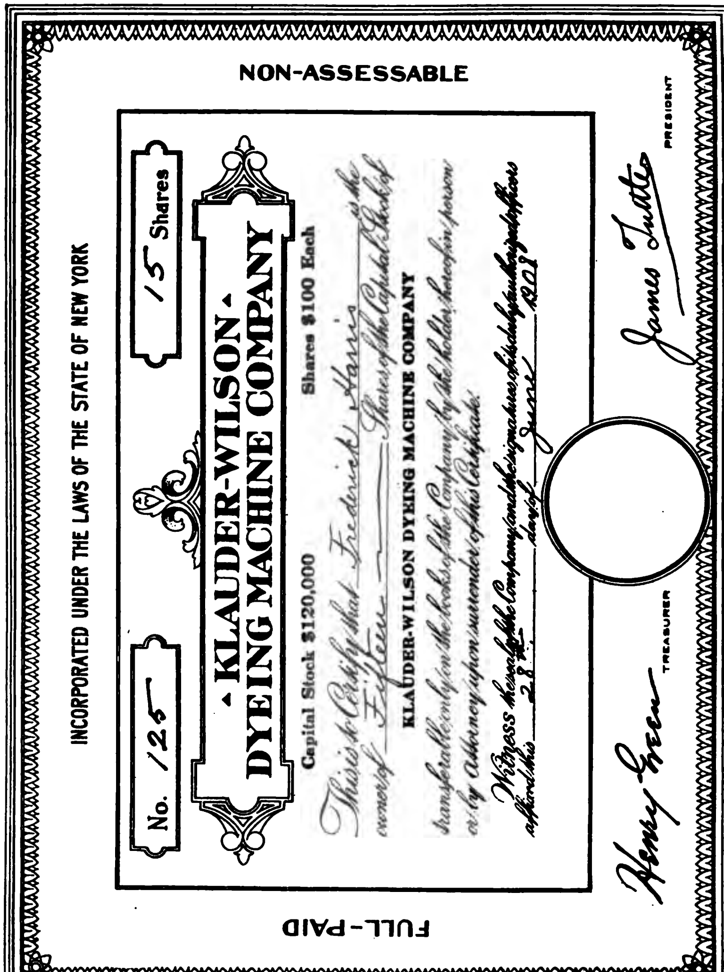
8. Make and solve five problems covering his expenses in Amsterdam.

9. Here he drew $1,000. How many gulden did he receive?

Make and solve five problems covering his expenses in Vienna.

The French standard is used in several other countries, forming what is known as the Latin Union. Among them are Italy, Greece, Switzerland, Spain, and Belgium.

10. Antonio Capelli wants to send money to a brother in Italy to pay the expenses of his coming to America. He goes to the banking house of Morosini and Company and buys a draft on the Bank of Italy at Rome. He gives the bank $150. After deducting 2 % exchange, what is the amount of the draft in francs?


NON-ASSESSABLE

INCORPORATED UNDER THE LAWS OF THE STATE OF NEW YORK

No. 125

15 Shares

KLAUDER-WILSON DYEING MACHINE COMPANY

Capital Stock $120,000 Shares $100 Each

This is to Certify that Frederick Harris is the owner of Fifteen Shares of the Capital Stock of

KLAUDER-WILSON DYEING MACHINE COMPANY

transferable only on the books of the Company by the holder hereof in person or by Attorney upon surrender of this Certificate.

Witness the seal of the Company and the signatures of its duly authorized officers affixed this 28th day of June 1908

Henry Green
TREASURER

James Tuttle
PRESIDENT

FULL-PAID

STOCKS

TERMS DEFINED

247. If several people desire to do business together, they may form a **company.** All states have laws governing the formation of companies. A company formed under such laws is said to be incorporated and is called a **corporation.**

Any number of people may belong to a corporation, each owning a part of its property. Large corporations, such as those that own railroads, have often many thousand members.

Money invested in business by a person or a corporation is called **capital.**

The capital of a corporation is called **stock.** It is divided into **shares.** A share may represent any amount. The most common value of a share is $100.

248. A person who invests money in a corporation receives one or more **certificates of stock,** indicating the number of shares he owns.

249. The amount named in the certificate is called its **face** or **par value.**

250. If the corporation makes a profit, each stockholder receives as his share of the profit a percentage of the face value of his stock. Such percentage is called a **dividend.**

251. The act of a corporation in deciding to give a dividend is called **declaring a dividend.** Sometimes when a business has paid a good dividend, the corporation does not pay it out, but uses the gains and profits to enlarge the business or to make improvements. For this reason an investor in stock may not get returns for his money for several years, and yet it may be absolutely safe.

252. The business of a corporation is managed by a **Board of Trustees** or **Directors,** elected by the stockholders, each share being entitled to one vote.

253. The value of a share of stock does not depend upon its original cost, or its face value, so much as upon the profits of the business, as shown by the dividends.

Stocks are sold sometimes for much more and sometimes for much less than their face value. The prices of stocks often change from day to day. The prices of many stocks are published daily. Such publications are called **quotations.**

254. When stocks are sold for more than the par value, they are sold "at a **premium**"; when for less than par value, "at a **discount.**"

Stocks are quoted in dollars and fractions of a dollar, thus: "Erie R. R., $44\frac{5}{8}$" means that Erie R. R. stocks sell for $44.625 a share.

BUYING AND SELLING STOCKS

255. An agent who buys and sells stocks, bonds, etc., is called a **broker.**

256. The commission, or **brokerage**, is always reckoned on the par value of the stock.

The commission charged by brokers is $\frac{1}{8}$ of one per cent, or $12\frac{1}{2}$¢ on a share, par value $100.

WRITTEN EXERCISES

257. Find the brokerage on the following:

1. 9500 shares Reading R. R. bought for $139\frac{1}{2}$, sold for $137\frac{5}{8}$.
2. 900 " Met. St. Ry. " " $114\frac{5}{8}$ " " $117\frac{3}{4}$.
3. 500 " N. Y. Dock " " 45 " " $47\frac{1}{4}$.
4. 600 " Amal. Copper " " $112\frac{1}{4}$ " " 110.
5. 8000 " Gt. North. Pf. " " 323 " " $327\frac{1}{2}$.

6. When a stock is quoted at 107, how much does a share cost? At 95? At $98\frac{1}{2}$? At par?

7. What was the cost of:

(1) 30,300 shares of Chicago, Milwaukee & St. Paul at 119?
(2) 1,025 " " Chicago & Northwestern " $147\frac{1}{2}$?
(3) 3,300 " " Colorado Fuel & Iron " $20\frac{7}{8}$?
(4) 1,250 " " Delaware & Hudson " $158\frac{1}{4}$?
(5) 2,800 " " Erie " $15\frac{1}{4}$?
(6) 200 " " Illinois Central " $128\frac{1}{2}$?
(7) 1,200 " " Louisville & Nashville " $98\frac{1}{2}$?
(8) 300 " " Mexican Central " $18\frac{1}{4}$?
(9) 6,700 " " New York Central " $100\frac{7}{8}$?

Add to each of the above $\frac{1}{8}$ of 1 % brokerage.

8. Mr. Melvin bought, through his broker, 250 shares of U. S. Steel at $35\frac{7}{8}$, and sold it at $41\frac{1}{4}$. How much did he make?

SOLUTION:

\$35.875 + \$.125 (commission) = \$36.00, cost of 1 share.
\$41.25 − \$.125 (commission) = \$41.125, selling price of 1 share.
\$41.125 − \$36.00 = \$5.125, gain on one share.
\$5.125 × 250 = \$1281.50, gain on transaction.

9. What is the annual income from 50 shares of stock, par value \$100, paying $1\frac{1}{2}$ % quarterly?

10. I invested \$1000 in stock, paying the par value of \$25 a share. How many shares did I buy? How much did each share cost me? (No brokerage was paid.)

11. The stock earned a dividend of $2\frac{1}{4}$ % semiannually. What was my annual income from it?

12. Mr. Tibbits bought 300 shares of C. M. & St. P. stock at $119\frac{3}{8}$. What did he pay, including brokerage?

13. The stock paid a quarterly dividend of 2 %. How much did he receive annually? What rate was this on his investment?

14. Mr. Alberts invested, through his broker, in 140 shares of U. S. Steel stock at 34. How much did it cost him?

15. If they pay $2\frac{1}{2}\%$ semiannually, what income will his stock produce? What per cent is this on his investment?

258. The following are some of the quotations of the New York stock market for one day:

CONSOLIDATED EXCHANGE SALES

		Open	High	Low	Last
23,830	Amal. Cop.	$57\frac{3}{4}$	$57\frac{3}{4}$	$55\frac{1}{4}$	$55\frac{7}{8}$
230	Am. Cen. Fnd.	$30\frac{1}{4}$	$30\frac{1}{4}$	$29\frac{1}{2}$	$29\frac{1}{2}$
170	Am. Ice	18	$19\frac{1}{8}$	$17\frac{3}{4}$	18
90	Am. Locom.	37	$37\frac{1}{2}$	$36\frac{3}{4}$	$36\frac{3}{4}$
23,120	Am. Smelt.	69	$69\frac{1}{8}$	$66\frac{3}{4}$	$67\frac{1}{8}$
2,110	Am. Sugar	$119\frac{1}{2}$	$121\frac{1}{2}$	$118\frac{3}{4}$	$119\frac{1}{8}$
1,770	Anacon. Cop.	$37\frac{1}{2}$	$37\frac{3}{8}$	36	$36\frac{1}{8}$
860	A. T. & S. F.	$73\frac{7}{8}$	74	72	$72\frac{1}{8}$
190	Balt. & Ohio	82	$82\frac{1}{4}$	$80\frac{5}{8}$	$80\frac{5}{8}$
10,730	Brook. R. T.	46	47	45	$45\frac{1}{4}$
140	Can. Pac.	146	$146\frac{1}{2}$	145	146
30	Ch. & Gt. W.	5	5	$4\frac{3}{4}$	$4\frac{3}{4}$

Make and solve ten problems on the above.

COMMON AND PREFERRED STOCK

259. Many corporations issue two kinds of stock, called **common** and **preferred**. The difference is in the obligations as to dividends. Preferred stock is guaranteed to pay a specified dividend, if there is any profit with which to pay dividends. This must be paid before common stock can receive any dividend. But preferred stock can never receive more than the amount specified, while the common can get all the surplus profit, if any.

WRITTEN EXERCISES

260. Solve:

1. A publishing company was incorporated with $100,000 capital, in 1,000 shares. Of these, 250 shares were preferred 6% shares. The remainder were common. At the end of the second year, the common stock drew 3%. How much was paid in dividends in all?

2. At that rate how much more would a share of preferred stock be worth than a share of common?

3. Later the common stock paid 4% semiannually. At that rate, how much more would a share of common be worth than a share of preferred?

4. Following is a list of quotations of railroad stocks made at two different times. At the rate of dividend mentioned, what per cent will a purchase of a share of each stock yield at each price quoted?

RAILWAY SHARES

Name	Yearly Dividend Per Cent	First Price	Last Price
(1) Atchison	6	$110\frac{1}{2}$	$72\frac{7}{8}$
(2) Atchison, pf.	5	106	85
(3) Baltimore & Ohio	6	$125\frac{1}{8}$	$80\frac{5}{8}$
(4) Baltimore & Ohio, pf.	4	$99\frac{1}{2}$	82
(5) Central of New Jersey	8	$239\frac{7}{8}$	170
(6) Chesapeake and Ohio	1	$65\frac{5}{8}$	$29\frac{1}{4}$
(7) Chi., Mil. & St. Paul	7	$199\frac{5}{8}$	$118\frac{7}{8}$
(8) Chicago & Northwestern	7	240	146
(9) Delaware & Hudson	9	$234\frac{3}{4}$	154
(10) Denver & Rio Grande, pf.	5	$91\frac{1}{2}$	49
(11) Great Northern, pf.	7	348	$121\frac{1}{4}$
(12) Illinois Central	7	$184\frac{1}{2}$	125
(13) Louisville & Nashville	6	$156\frac{1}{2}$	$96\frac{1}{4}$

5. Mr. Butler needed $6,000 to use in his business. He gave his note for $6,000 to the Liberty Bank, for 90 days, at 5%. What was the interest?

6. He deposited as security with his note 100 shares of N. Y. Central stock, quoted at 102. What was the par value of the security? The market value?

7. The bank discounted the note; that is, deducted the interest from the $6,000. The balance was placed to Mr. Butler's credit in the bank. How much was added to his bank account?

8. When the note was due, Mr. Butler gave a new note for $3,500, for 90 days, and his check for the balance and the interest on the new note. What was the amount of his check?

9. When the note was due, Mr. Butler, having met losses, was unable to pay the note, and the bank refused to renew it. They then sold enough of the security to pay the note. This was at that time quoted at 96.

(1) How many shares were sold?

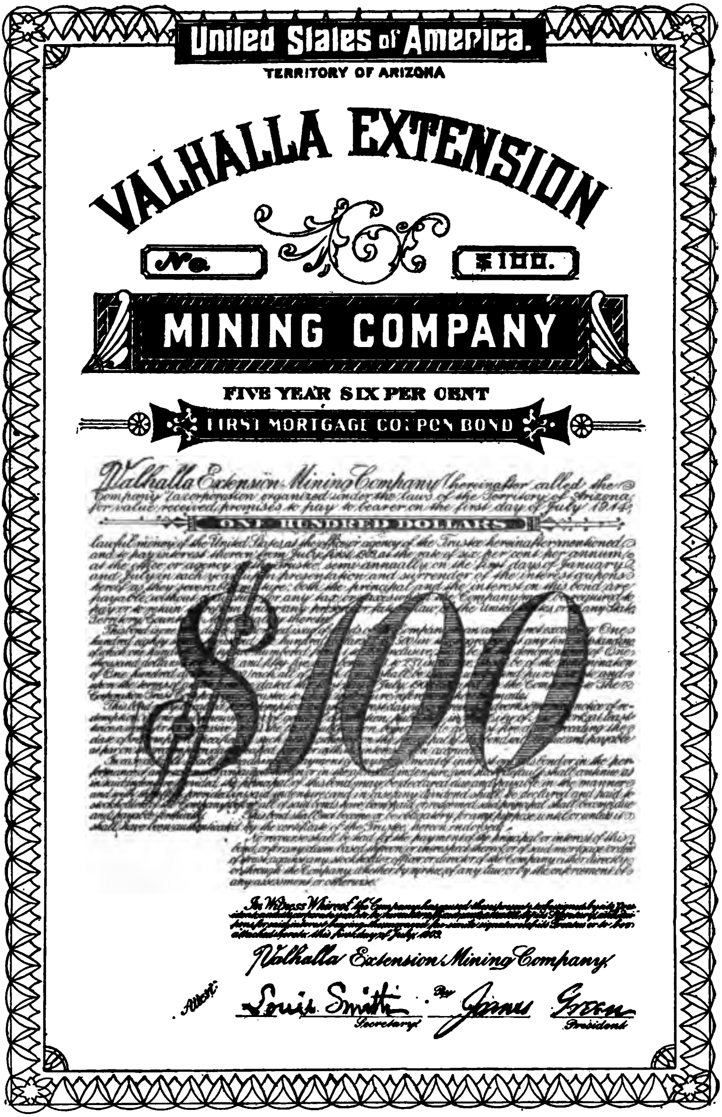
(2) What was the par value of the stock returned to Mr. Butler?

(3) How much did he lose by the decline in value of the stock?

BONDS

261. Corporations, as well as individuals, borrow money to meet the needs of business. Sometimes the corporation can raise money by simply giving its note signed by the officers of the company. But usually security is required.

The most usual form of security for a corporation to give is a **bond.** A bond is really a mortgage on the property of the corporation, empowering the purchasers of the bond, who are the actual money loaners, to take possession of the property if the corporation fails to meet its obligations.



United States of America.

TERRITORY OF ARIZONA

VALHALLA EXTENSION

No. $100.

MINING COMPANY

FIVE YEAR SIX PER CENT

FIRST MORTGAGE COUPON BOND

Valhalla Extension Mining Company (hereinafter called the Company) a corporation organized under the laws of the Territory of Arizona, for value received, promises to pay to bearer on the first day of July 1914;

ONE HUNDRED DOLLARS

lawful money of the United States at the office or agency of the Trustee hereinafter mentioned, and to pay interest thereon from July first 1909 at the rate of six per cent per annum, at the office or agency of the Trustee, semi-annually on the first days of January and July in each year upon presentation and surrender of the interest coupons hereto as they severally mature; both the principal and the interest on this bond are payable without deduction for any tax or taxes which the Company may be required to pay or to retain [illegible]

[illegible]

This bond shall not become or be obligatory for any purpose unless and until it shall have been authenticated by the certificate of the Trustee hereon endorsed.

No recourse shall be had for the payment of the principal or interest of this bond, or for any claim based hereon or in respect hereof or of said mortgage or deed of trust, against any stockholder, officer or director of the Company, either directly or through the Company, whether by virtue of any law or by the enforcement of any assessment or otherwise.

In Witness Whereof the Company has caused these presents to be signed by its President [illegible] this first day of July, 1909.

Valhalla Extension Mining Company

Attest: Louis Smith, Secretary. By James Green, President.

Bonds of large corporations, like stocks, are sold on the market and are quoted daily.

262. Stocks are certificates of ownership of a share in the property of a corporation. Their returns, in the form of dividends, depend wholly upon the success of the business of the corporation. The returns can be paid only after all expenses have been met.

263. Bonds are certificates of a share in a mortgage on the corporation's property. Their returns, in the form of interest, are fixed in the bond, and must be paid before dividends are paid.

Governments also, national, state, county, and city, raise money by issuing bonds, which are a mortgage on the property of the government.

Most bonds have **coupons** attached to them, which are cut off and presented as proofs that interest is due.

WRITTEN EXERCISES

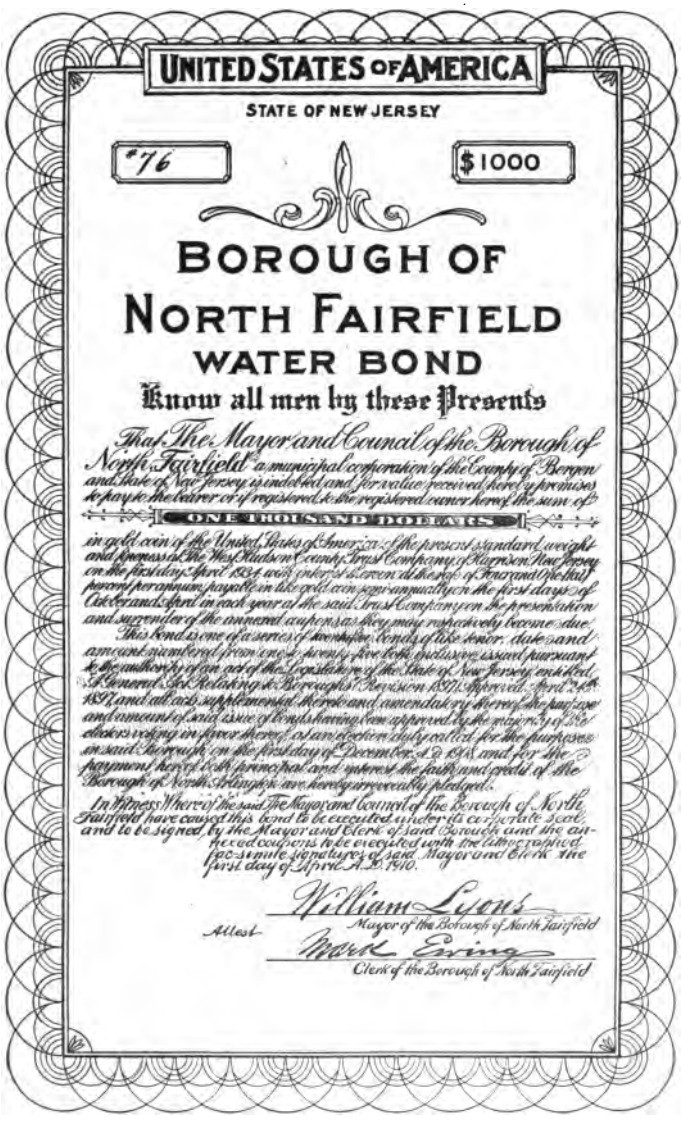
264. Solve:

1. If U. S. 3% coupon bonds are quoted at $101\frac{5}{8}$, how many can a bank purchase with $100,000? What will be the annual income from the investment?

2. If Japanese Imperial Government $4\frac{1}{2}$% bonds are quoted at 86, how many can be purchased for $24,000?

3. What will be the annual income from the investment? What rate of interest on the price will that represent?

4. Which is the better investment; New York City $4\frac{1}{2}$% bonds at $102\frac{3}{8}$, or B. & O. Railway 4% bonds, at $99\frac{1}{2}$? How much per bond?



UNITED STATES OF AMERICA

STATE OF NEW JERSEY

No. 76 $1000

BOROUGH OF NORTH FAIRFIELD

WATER BOND

Know all men by these Presents

That The Mayor and Council of the Borough of North Fairfield a municipal corporation of the County of Bergen and State of New Jersey is indebted and for value received hereby promises to pay to the bearer or if registered to the registered owner hereof the sum of

ONE THOUSAND DOLLARS

in gold coin of the United States of America of the present standard, weight and fineness at The West Hudson County Trust Company of Harrison New Jersey on the first day of April 1934 with interest thereon at the rate of four and one half per cent per annum payable in like gold coin semi annually upon the first days of October and April in each year at the said Trust Company upon the presentation and surrender of the annexed coupons as they may respectively become due.

This bond is one of a series of twenty five bonds of like tenor, date and amount numbered from one to twenty five both inclusive issued pursuant to the authority of an act of the Legislature of the State of New Jersey entitled "A General Act Relating to Boroughs" Revision 1897 approved April 2nd 1897 and all acts supplemental thereto and amendatory thereof the purpose and amount of said issue of bonds having been approved by the majority of the electors voting in favor thereof at an election duly called for the purpose in said Borough on the first day of December A. D. 1909 and for the payment hereof both principal and interest the faith and credit of the Borough of North Fairfield are hereby irrevocably pledged.

In Witness Whereof the said the Mayor and Council of the Borough of North Fairfield have caused this bond to be executed under its corporate seal and to be signed by the Mayor and Clerk of said Borough and the annexed coupons to be executed with the lithographed fac-simile signatures of said Mayor and Clerk the first day of April A. D. 1910.

William Lyons
Mayor of the Borough of North Fairfield

Attest

Mark Ewing
Clerk of the Borough of North Fairfield

INSURANCE

TERMS DEFINED

265. **Insurance** is a form of provision against loss from certain causes, such as fire, death, shipwreck, and various accidents.

An **insurance policy** is an agreement to pay to the loser, in case of loss from a specified cause, a sum either stated in the policy or to be determined on some fixed plan.

The price paid for insurance is called the **premium.**

The price or **rate** of insurance is usually stated as such a per cent on $100 or $1000.

There are two classes of insurance, — **property insurance** and **personal insurance.**

PROPERTY INSURANCE

266. The most common forms of property insurance are **fire** insurance, **marine** insurance, **burglar** insurance, and **plate glass** insurance. There are various other kinds, however.

PERSONAL INSURANCE

267. Common forms of personal insurance are **life insurance, accident insurance,** and **credit insurance.**

Of these the most important is life insurance.

268. **Life insurance** is a form of contract in which the insurer, a life insurance company, agrees to pay a fixed amount upon the death of the insured person. The person to receive the payment is called the **beneficiary.**

The terms of the contract, including the name of the beneficiary, are set forth in a document called the **policy.**

Sometimes it is specified in the policy that, if the insured live to a fixed date, the money shall be paid to him.

The insured person pays the company a specified amount, in most cases annually in advance. This is called a **premium.**

In some policies the premium is to be paid during the life of the insured. In others it is to be paid for a definite number of years, as ten or twenty.

The amount of the premium varies according to the age of the insured when the policy is taken out and the number of years during which payments are to be made.

In the following tables rates are given for ages from 21 to 35 for payments during life, and for a 20-year period.

Observe the gradual increase of the premiums with the increase of the age. Note also the difference between the annual premiums of the twenty-year policy and those of the "straight life" policy.

PREMIUM FOR $1000 INSURANCE

ANNUAL PAYMENTS

AGE	DURING LIFE Premium	DURING TWENTY YEARS Premium
21	$15.91	$23.49
22	16.26	23.86
23	16.62	24.26
24	17.00	24.67
25	17.40	25.09
26	17.83	25.55
27	18.29	26.02
28	18.76	26.51
29	19.27	27.03
30	19.80	27.56
31	20.36	28.12
32	20.94	28.70
33	21.57	29.30
34	22.53	29.93
35	22.92	30.39

WRITTEN EXERCISES

269. Solve:

1. Mr. Weathesby owns two houses. One is built of fire-proof material throughout. The other is frame. He insured the fire-proof house for $5,000 and its contents for $3,000, at a premium of $3.50 per $1,000 per year. The frame house he insured for $6,500 at a rate of $7.50 per $1,000. What was the premium on each?

2. Bullard Brothers insured their brick store for $50,000 at $12 per $1,000. Their stock they insured against loss by fire for $75,000 at $10.50 per $1,000, the insurance on the store and stock being divided equally among three companies. They also carried burglar insurance for $25,000 at $6 per $1,000. Their plate glass windows they insured for $500 at $1.75 per $100. What was their total annual expenditure for insurance premiums?

3. The building was gutted by fire. Much of the stock was injured or destroyed, and, during the fire burglars carried away a case of diamonds. The *adjusters* determined that the loss on the building was 75 % of the amount insured, on the stock by fire 45 %, and by burglary $4,500. The windows were a total loss. How much did each company lose?

4. Mr. Johnson, when 30 years old, took out a life insurance policy for $5,000, on the 20 annual payment plan, naming Mrs. Johnson as beneficiary. What were his annual premiums?

5. After 12 years he died. How much had he paid in premiums?

6. How much did Mrs. Johnson receive in excess of the premiums paid?

Make and solve ten life insurance problems.

REVIEW

270. Solve:

1. Mr. Lane gave Mr. Berry his note for $2,000 for four months with interest at 6 %. At the end of the four months how much was due Mr. Berry?

2. If Library Bureau stock is quoted at 120, what will be the cost of 23 shares?

3. If this stock pays 8 %, what is the rate of income?

4. Mr. White bought 150 shares of Illinois Central Railway stock at 111. In two weeks, the price of the stock had risen to 119, and he sold his shares. He paid his broker $\frac{1}{8}$ % for buying and $\frac{1}{8}$ % for selling. How much did he gain?

5. At the age of 25 Mr. Day had his life insured for $2,000, taking out a twenty-year policy, the rate for which was $25.09. At the end of the 20 years, he received the $2,000. How much more was this than he had paid in?

6. How much will Mr. Daniels receive at the bank on a three months' note for $600, the interest being at 6 %?

7. Write an interest-bearing note at 5 %. What amount must be paid when the note falls due?

8. A man had a house which cost him $3,480. He had it insured for $\frac{3}{4}$ of this amount, at a cost of $6.40 per thousand. The insurance ran out and was not renewed. Three days afterward the house was totally destroyed by fire. How much greater was the man's loss than it would have been if the house had been insured at the time of the fire?

9. A man has $12,000 which he wishes to invest. Will it be better for him to buy 6 % stock at 120, or 5 % stock at 110? How much better?

10. When Delaware & Hudson R. R. stock is quoted at 225, how many shares can be bought for $11,500? What is the rate of income if this is 7 % stock?

11. Mr. James invested $27,390 in an 8 % stock which he bought at 114, brokerage $\frac{1}{8}$ %. What was his yearly income from this stock?

12. Later he sold his stock at 118. What did he receive for it? How much did he gain by the transaction? What would have been his loss if he had sold it for 111?

13. How much greater was Mr. James' rate of income from this stock than the rate of income of the one to whom he sold it?

14. Which would you rather buy, 5 % stock at 75, or 7 % stock at 105? Why?

15. Find the proceeds of a 60-day note for $925, bearing interest at 6 %, discounted 24 days after it was made.

16. Mr. Brown bought a farm for $3,400. He paid $1,300 cash. For the balance he gave a mortgage with interest at 5 %. This mortgage was not paid until the end of 5 years. What was the entire amount, including interest, that he had to pay for the farm?

SUMMARY OF CHAPTER V

BANKING.

Banks.
Clearing Houses.
Savings Banks.
Trust Companies.
Loans.
Mortgages.
Bank Discount.
Exchange.

BUSINESS PRACTICE.

Stocks.
Bonds.
Insurance.

CHAPTER VI

HOW TO SOLVE A PROBLEM — ELEMENTS OF ALGEBRA

THE THREE METHODS

271. The ratio of 12 to 6 = the ratio of 8 to what number?

Read the problem carefully, to see

(1) What is given?

There is given a ratio, 12 : 6, which is 2, and one term, 8, the dividend of an equal ratio.

(2) What is to be found?

There must be found the second term, or the *divisor*, of the equal ratio.

(3) What is their relation?

The ratio is the quotient of one term divided by the other. 8, the first term, is the dividend. 2 is the ratio or quotient. The relation is that of dividend to quotient.

(4) What principle must you employ to find the unknown term from the known?

The principle is this:

Dividend ÷ quotient = divisor.

$8 \div 2 = 4$ = the divisor or second term of the incomplete ratio.

That is, $12 : 6 = 8 : 4$.

How can you prove the correctness of your answer? Divisor × quotient = dividend. $4 \times 2 = 8$.

Give other proofs.

Name the steps in the solution of a problem.

Select ten problems in your book and name these steps for each one.

Naming the steps in the solution of a problem is called **stating the problem.** It is important to state all problems before beginning to solve them.

Several methods of solving problems are used in arithmetic. One is **analysis,** sometimes called **unitary analysis.** Another makes use of **ratio,** and a third employs the **equation.** Frequently the equation includes one or both of the other methods.

272. SOLVING BY ANALYSIS

1. If 2 apples cost 4 cents, what will 6 apples cost?

2 apples cost 4 cents.
1 apple costs $\frac{1}{2}$ of 4 cents = 2 cents.
6 apples cost 6 × 2 cents = 12 cents.

2. What are three eggs worth at 24 cents a dozen?

If 12 eggs cost 24 cents, 1 egg will cost $\frac{1}{12}$ of 24 cents = 2 cents. 3 eggs will cost 3 × 2 cents = 6 cents.

3. $\frac{3}{4}$ of Daisy's money is \$15. How much money has she?

$\frac{3}{4}$ of the money is \$15.
$\frac{1}{4}$ of the money is $\frac{1}{3}$ of \$15 = \$5.
$\frac{4}{4}$ of the money is 4 × \$5 = \$20.

273. SOLVING BY RATIO

1. If 2 apples cost 5 cents, what will 6 apples cost?

The ratio of 6 apples to 2 apples is 3.
Therefore, 6 apples cost 3 × 5 cents = 15 cents.

2. What are 3 eggs worth at 24 cents a dozen?

The ratio of 3 : 12 = $\frac{1}{4}$; $\frac{1}{4}$ of 24 cents = 6 cents.

3. $\frac{3}{4}$ of Ruth's money is \$20. How much money has she?

The ratio of $\frac{4}{4} \div \frac{3}{4} = \frac{4}{3}$. $\frac{4}{3}$ of \$20 = $\$\frac{80}{3}$ = \$26.66$\frac{2}{3}$.

WRITTEN EXERCISES

274. Solve the following by any method:

1. If $\frac{5}{8}$ of an acre of land was sold for \$1,200, what is the value of $2\frac{1}{2}$ acres at that price?

2. If $\frac{3}{8}$ of Mr. Lanning's crop of wheat is worth \$840, what is the value of the crop?

3. If $12\frac{1}{2}$ yards of silk cost \$20, how much will 100 yards cost? How many yards can be bought for \$80? For \$95? For \$105?

4. A jobber bought 100 books for \$16. What was the cost of 75 books at that rate?

5. A farmer had 1,000 bushels of potatoes. He sold 750 bushels for \$810. What is the value of his crop at that rate?

6. Mr. Rankin deposited \$2,400 in the bank, which was $\frac{3}{4}$ of his money. How much had he?

7. A farmer sold $37\frac{1}{2}$ pounds of butter for \$7.50. What are 100 pounds worth at that rate?

8. If $62\frac{1}{2}\%$ of a number is 15, what is the number?

9. $\frac{5}{6}$ of a number is 80. What is the number?

10. $87\frac{1}{2}\%$ of my bank account is \$3,500. What is $12\frac{1}{2}\%$ of it?

11. $62\frac{1}{2}\%$ of Mr. Randolph's wheat is 2,500 bushels. He sold $37\frac{1}{2}\%$. How many bushels did he sell?

12. If $\frac{5}{8}$ of a barrel of apples are worth \$2, what are $\frac{3}{10}$ of a barrel worth?

13. A real estate agent sold a house for \$4,200 and made 40% on it. What did it cost?

14. He sold another for the same price and lost 40%. What did it cost?

15. What was the gain or loss on both sales?

SOLVING BY EQUATIONS—THE USE OF LETTERS

275. A still better method for the solution of different problems is by the use of the equation, using a letter or letters, as x, or y, to represent the quantity to be found.

Numbers are often represented by letters of the alphabet instead of by the ordinary numerals. Numbers represented by letters are called **literal quantities.**

Thus we may speak of a apples, b boys, x hats.

We can use the letters in calculation as if they were figures. They may represent any numbers that we choose and then may be added, subtracted, multiplied, or divided, as we please.

Suppose you say that $a = 6$, then 6 boys may be written a boys.

2×6 boys may be written $2 \times a$ boys.

Suppose you decide that you will use b for 2. Then 2×6 boys may be written $a \times b$ boys, etc.

2 boys + 6 boys will be a boys + b boys.

In writing numbers by the use of letters, use the same signs that you do with numerals. In representing multiplication, however, the sign may be omitted. $ab = a \times b$; $2a = 2 \times a$; $2bc = 2 \times b \times c$. All other signs must always be used to indicate the operations to be performed.

In the expression, $2abc$, 2 is called the **coefficient** of abc.

When a literal quantity is represented as multiplied by an Arabic numeral, or figure, the figure is called the **coefficient** of the literal quantity.

276. Letting $a = 2$, $b = 3$, $c = 4$, and $d = 5$, find the values of:

1. $a + b + c$.
2. $2b - d$.
3. $3a + 2b - 2c$.
4. $2d + a - 3b$.
5. $ab + cd$.
6. $cd - b$.
7. $2bc$.
8. $3abc$.

277. POSITIVE AND NEGATIVE QUANTITIES

$5+4-4=$ $3+2-2=$

$6-5+5=$ $7-6+6=$

In each of the above examples the amount added equals the amount subtracted. $+4$ and -4 offset each other. So do -5 and $+5$; $+2$ and -2; -6 and $+6$.

278. 1. $5+3-4=$ 5. $7-7+8=$

2. $4+6-5-3=$ 6. $5-3+2+3=$

3. $5-4+3-2=$ 7. $4-6+8-9+17-20.$

4. $8-6+10-11=$ 8. $6+6+6-7-7-7-8.$

In each of the above we can perform each operation separately; or we can find the result by subtracting the sum of the numbers having the *minus* sign from the sum of the numbers having the *plus* sign. (A quantity having neither the + sign nor the − sign is supposed to have the + sign.) Thus, in example 2, we can say $4+6=10$; $10-5=5$; $5-3=2$. Or, we can say $4+6=10$; the sum of the minus quantities $=8$; $10-8=2$.

Quantities having no sign, or the sign +, are called **positive** quantities.

Quantities written with the sign − are called **negative** quantities.

Sometimes the result of combining the positive and negative terms of a quantity is less than nothing; that is, it is negative.

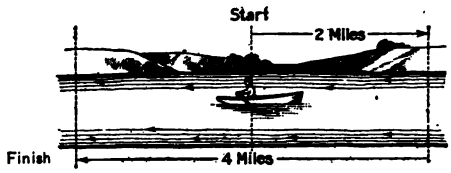
A man at the end of a year found that money was due him to the amount of $2526 and he owed debts amounting to $3186. What was the excess of his debts over his credits?

Write the difference with a minus sign as a negative quantity. Thus: $2526-3186=-660$.

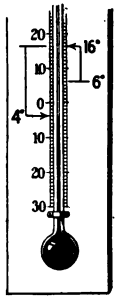
WRITTEN EXERCISES

279. A boy started to row his boat up the stream at the rate of 2 miles an hour. The stream flowed down at the rate of 4 miles an hour. At the end of an hour was he above or below his starting place, and how far?

He had rowed up 2 miles and been carried down 4 miles, thus:



At the end he was 2 miles below the starting place. The answer, then, is a negative quantity, -2. That is, it could be represented by an equation, $2-4=-2$. His course up the stream had been 2 less than nothing.



One morning the thermometer showed 6 degrees above zero. During the day the mercury rose 10 degrees, and at night fell 20 degrees. How far below zero was it at night? Write this as a negative quantity.

$$6+10-20=-4$$

Write as negative quantities the results of the following:

1. $6-9=$
2. $2+3-4-4=$
3. $8-1+9-20=$
4. $-3-4-5+8=$
5. $6+3-7-4-8=$
6. $-1+2-3+1=$
7. $5+6-7+8-9+16-20=$
8. $10-100-90+675-840=$
9. $56+75+173-1000-20=$

280. LITERAL QUANTITIES

2 boys + 2 boys = 4 boys.	2×2 girls = 4 girls.
$2a + 2a = 4a$.	$2 \times 2a = 4a$.
3 books − 2 books = 1 book.	6 horses ÷ 2 = 3 horses.
$3b - 2b = 1b$.	$6b \div 2 = 3b$.

What is the numeral written before a literal quantity called?

A literal quantity having no coefficient is understood to have 1 for a coefficient. $b = 1b$. $c = 1c$.

As letters stand for quantities, they can be added, subtracted, multiplied, and divided like the quantities they stand for.

WRITTEN EXERCISES

281. Perform the operations indicated by the signs in the following problems:

1. $2a + 3a =$	**7.** $2d + 5d - 6d =$
2. $3b - b =$	**8.** $2c - 4c =$
3. $b - 3b =$	**9.** $5c - 6c =$
4. $6a - 4a =$	**10.** $18e - 21e + 6e =$
5. $15b - 12b =$	**11.** $7f - 8f + 9f - 10f =$
6. $3c - 4c =$	**12.** $2a + 3a - 4a - 2a =$

ADDITION

282. Add 2 dogs and 3 cats.

The answer is merely 2 dogs + 3 cats.

You cannot state your answer in a single term.

Add a and b.

The answer is necessarily just $a + b$.

The sum of a and b, less c, $= a + b - c$.

Unlike literal quantities to be added or subtracted are merely written with the signs to show the operation.

Add: a, b, and c.

Add: $a + b + 2b$. $b + 2b = 3b$. *Ans.* $a + 3b$.

Add: a, $2a$, $3a$, $2b$, $3b$.

These terms may be arranged in this way:

SOLUTION:

$$\begin{array}{r} a \\ 2a + 2b \\ 3a + 3b \\ \hline 6a + 5b \end{array}$$

Add: $2a$, $-3a$, $-b$, $2b$.

SOLUTION:

$$\begin{array}{r} 2a - b \\ -3a + 2b \\ \hline -a + b \end{array}$$

Add:

(1)	(2)	(3)	(4)
$2b$	$-3c$	$-d$	x
$-b$	c	$3d$	$-4x$
$3b$	$4c$	$5d$	$5x$
$-2b$	$-5c$	$-8d$	$-10x$
$4b$	$6c$	$10d$	$6x$

In adding algebraic quantities place like quantities in the same columns, with the proper signs, and add.

283. Add: (1)

$$\begin{array}{r} 2a + 3b - 3c \\ 3a + 3b - 2c \\ \hline \end{array}$$

(2)

$$\begin{array}{r} 6b - 7c - 8a \\ 3b - 2c - 9a \\ \hline \end{array}$$

(3)

$$\begin{array}{r} 5x + 3y - z \\ 2x - 6y - 3z \\ \hline \end{array}$$

(4)

$$\begin{array}{r} 7m + 3n - p - 2 \\ 14m - 6n + p - 7 \\ \hline \end{array}$$

(5)

$$\begin{array}{r} 2c - d + x \\ c \quad -2x \\ a + 3c + d - 5x \\ \hline \end{array}$$

How do you add two quantities, one positive and one negative?

WRITTEN EXERCISES

284. Arrange the following quantities in columns and add:

6. $2b - 3 + 4c - 6b - 5c + b - c + 5.$
7. $6a - 7c + 2d - 5c + 6d - a + 2a - 7d + 6c.$
8. $8x - 5x - 3x + 2y - 7c - 6y - 4y + 8y - 24.$
9. $15 + 20 + 2a - 3a - 6d + 4a - 5d + c + 6d - 20c.$
10. $2z - x + 3y - 6z - y - 6x + 8y - 6 - z + 3 - 5z + x.$
11. $2ab + 2ac + 2d - x - 3ab - 4d + 3ac + 6x.$
12. $4xy + 4xz - 2y + 2z + 5xy + 6xy - 3xz - 2xz.$
13. $y + x + xy + z + xyz + 2xy + 3xyz + 2y + 3x + 4z.$
14. $abc + bcd + cde + 2abc + 3cde + 4bed + 5 + a + 2a.$
15. $20xy + 15x + 30y + 40x + 6xy + 7xy - x - y - 2x.$
16. $3abx - 2by + 4cd - 2abx - 4abx + 2by - 54cd.$
17. $x + y - z + a - z + x + a - y.$
18. $15xyz - 7abc - abe - xyz - xy + ab + ax - 6d.$

SUBTRACTION

285. John had \$15. He owed \$20.

Represent his condition as a negative quantity.

$$15 - 20 = -5.$$

He received \$25. How much was he now worth?

$$-5 + 25 = +20.$$

That is, you add 25 to his credits.

Subtract $2x$ from $3x$.	$3x - 2x = x.$
Subtract 2 from 1.	$1 - 2 = -1.$
Subtract $3x$ from $2x$.	$2x - 3x = -x.$
Subtract $-2x$ from $3x$.	$3x - (-2x) = +5x.$
Subtract $-4ab$ from $6ab$.	$6ab - (-4ab) = 10ab.$
Subtract $-3mn$ from $6mn$.	$6mn - (-3mn) = 9mn.$

Add:	Add:	Subtract:
-5	-4	-4
$+25$	$+12$	-12
$+20$	$+8$	$+8$

Subtracting a negative quantity is the same as adding a positive quantity, and subtracting a positive quantity is the same as adding a negative quantity.

To subtract one quantity from another, change the sign of the subtrahend and proceed as in addition.

WRITTEN EXERCISES

286. Subtract:

(1)	(2)	(3)	(4)	(5)
-30	-2	$-2a$	$-6b$	$-20abc$
12	16	$5a$	$4b$	$45abc$

(6)	(7)	(8)	(9)	(10)
-5	16	-18	17	20
-3	-7	-6	-18	-30

(11)	(12)	(13)	(14)	(15)
-40	16	-17	30	$-16y$
-35	-2	-20	-30	$12y$

16. $3x-5y+z-6$ and $2x-47+z-4$

17. $2ab+3cd-2b+a-6$ and $3ab-2cd+b-3a+10$

18. $9a-10b+3c+6abc$ and $10a+6b+5c-2abc$

19. $4x-5y+8z+14xy$ and $5x+10y-8z+5xy$

20. $4x+6y+z-8$ and $8x+2y-3z-4$

21. $15ab-2+13ac-4d$ and $6ab-4+7ac+8d$

22. $25d-13a+6bc-8b$ and $16d+14a+7bc-2b$

23. $20ab+30b-60cd-18$ and $10ab+40b-20cd+28$

MULTIPLICATION

287. Multiply:

$a \times 2$.	$a \times 2 = 2\,a$.
$a \times b$.	$a \times b = ab$.
$a \times b \times c$.	$a \times b \times c = abc$.
$2\,a \times 3\,b$.	$2\,a \times 3\,b = 6\,ab$.

When letters are written one after another, without a sign, multiplication is indicated.

WRITTEN EXERCISES

288. Write in different ways the products of:

1. a, b, and c.
2. b, e, and f.
3. b, e, and a.
4. x, y, and z.
5. a, b, c, d, e, and f.
6. d, c, and e.
7. $2\,a \times 2\,b = 4\,ab$.
8. $3\,a \times 4\,b =$
9. $5\,c \times 6\,d =$
10. $7\,c \times 5\,c =$

289. MULTIPLYING A QUANTITY BY ITSELF.

$2 \times 2 = 2^2 = 4$. $\qquad a \times a = a^2$.

A quantity, whether numeral or literal, when multiplied by itself is **squared**. If letters are used, the squaring is merely indicated by the sign. This sign is called an **index** (plural, *indices*). Every number has an index written or expressed. If no index is expressed, 1 is understood.

a is said to be in the first degree and its index, not expressed, is 1. It could be written a^1. a^2 is read "a square," or "a in the second degree." a^4 is read "a fourth power," or "a in the fourth degree."

Read: a; a^2; b^3; c^4; d^5.

$2 \times 2 = 2^2 = 4$. $\qquad a \times a = a^1 \times a^1 = a^{1+1} = a^2$.

$2^2 \times 2 = 2^3 = 8$. $\qquad a^2 \times a = a^{2+1} = a^3 = a \times a \times a$.

A quantity is squared by multiplying it by itself. It is cubed by multiplying it twice by itself. It is raised to the fourth power by multiplying it three times by itself.

To indicate the multiplication of a quantity by itself, add its indices.

WRITTEN EXERCISES

290. Multiply:

1. $2a \times a \times ab$.
2. $a \times b \times cd \times ab$.
3. $ax \times bx$.
4. $2c \times abc$.
5. $a \times 2b \times a^2c$.
6. $a^3 \times ab \times b^2$.
7. $xy \times x^2y^2 \times xyz$.
8. $abc \times 2a^2 \times 3b \times 3c^2$.

291. EXPRESSING MULTIPLICATION.

Multiply $2+3$ by 3.

SOLUTION: $(2+3)=5$; $5 \times 3 = 15$.

Or; $3 \times 2 = 6$; $3 \times 3 = 9$; $9+6=15$.

$$\begin{array}{r} 2+3 \\ \underline{\quad\quad 3} \\ 6+9=15. \end{array}$$

We can add the quantities and then multiply the sum; or we can multiply each quantity and add the products.

Multiply $a+b$ by 2.

$(a+b) \times 2 = 2a+2b$; or,

$$\begin{array}{r} a+b \\ \underline{\quad\quad 2} \\ 2a+2b \end{array}$$

$(a-b) \times 3 = 3a-3b$.

292. In using the signs of multiplication, or division, with quantities connected by $+$ or $-$, it is necessary to inclose these quantities in brackets ( ), to show that the entire quantity is to be multiplied or divided.

$(a+b)2$ means that the quantity $a+b$ is to be multiplied by 2; $a+b\,2$ would mean the b alone was to be multiplied.

What is the meaning of $(a-c) \div b$?

WRITTEN EXERCISES

293. Express by signs:

1. $x + y$ multiplied by 3.
2. $a + b + c$ multiplied by x.
3. $3a - 4 + c$ multiplied by $2c$.
4. $a - y$ multiplied by $c + d$.
5. $4a - 3$ multiplied by $3a - 4x$.
6. $2a - z$ divided by $a - b$.
7. $4 - 5x + 3y$ divided by $2 - 2a$.

8. The sum of a, b, and c divided by the difference of x and y.

294. MULTIPLICATION OF NEGATIVE QUANTITIES.

$2 \times -a = -2a$. $\quad a \times -a = -a^2$. $\quad 2 \times -a = -2a$.

If Mr. Jones owes \$10, it is represented algebraically as $-\$10$. If he owes \$10 to each of two pupils, it is represented algebraically as $2 \times -\$10$, or $-\$20$.

$$-a \times -a = +a^2.$$

In multiplication in algebra, the product of two negative quantities or of two positive quantities is positive; the product of a negative quantity and a positive quantity is negative.

WRITTEN EXERCISES

295. Multiply:

1. $a \times -a$. 2. $b^2 \times b^2$. 3. $-cd \times cd$. 4. $x^2 \times -x^3$.

296. MULTIPLYING QUANTITIES OF SEVERAL TERMS.

Multiply $a + b + 2c$ by 2.

$$\begin{array}{r} a + b + 2c \\ \hline 2 \\ \hline 2a + 2b + 4c \end{array}$$

$a + b - c - d$ by a.

$$\begin{array}{r} a + b - c - d \\ \hline a \\ \hline a^2 + ab - ac - ad \end{array}$$

In multiplying a quantity composed of numbers connected by + or −, multiply each number separately, giving the product in each case the proper sign.

297. WHEN THE MULTIPLIER HAS SEVERAL TERMS.

Multiply $a+b$ by $a+b$.

$$\begin{array}{r} a+b \\ a+b \\ \hline a^2+ab \\ ab+b^2 \\ \hline a^2+2ab+b^2 \end{array}$$

$x+y-z$ by $x+2y$.

$$\begin{array}{r} x+y-z \\ x+2y \\ \hline x^2+xy-xz \\ 2xy+2y^2-2yz \\ \hline x^2+3xy-xz+2y^2-2yz \end{array}$$

When the multiplier has more than one term, multiply each term of the multiplicand by each term of the multiplier and add the results.

WRITTEN EXERCISES

298. Multiply:

1. $a+b+c$ by $a+b$.
2. $x-2y-2z$ by $2z-2y$.
3. $m+n+p$ by $2-m$.
4. $a-b-2c+cd$ by $2a-3$.
5. $ab-ac-ad$ by $2ab-3ac$.
6. $3+x-y$ by $3+x-y$.

DIVISION

299. In division we reverse the process of multiplication.

Divide 4 by 2. $4 \div 2 = 2$.

Divide $4a$ by 2. $4a \div 2 = 2a$.

Divide a by b.

$a \div b = \frac{a}{b}$. The result can only be indicated.

$2^2 = 2 \times 2$; $2^2 \div 2 = 2 \times 2 \div 2 = 2$.

$a^2 = a \times a = a^{1+1}$.

How do we multiply literal quantities?

In dividing a^2 by a we subtract the indices; $2-1=1$.

$$a^2 \div a = a^{2-1} = a^1 = a.$$

Divide a by a. $a \div a = a^{1-1} = 1\,a^0 = 1$.

$$\frac{2\,c}{c} = 2\,c^{1-1} = 2\,c^0 = 2.$$

$$\frac{2\,a}{2\,b} = \frac{2}{2} \times \frac{a}{b} = \frac{a}{b}. \quad \textit{Ans.}$$

In dividing literal quantities having coefficients, the coefficients may be divided, while the division of the literal numbers is only indicated.

Divide $4\,a$ by $2\,b$. $4\,a \div 2\,b = \frac{\overset{2}{\cancel{4}}\,a}{\underset{1}{\cancel{2}}\,b} = \frac{2\,a}{b}$.

WRITTEN EXERCISES

300. Solve:

1. $\frac{2\,a^2}{2} =$
2. $\frac{2\,a^2}{a^2} =$
3. $\frac{36\,a}{4\,a} =$
4. $\frac{48\,b^2}{4\,b} =$
5. $\frac{50\,c^3}{10\,c^2} =$
6. $\frac{2\,ab}{2\,a} =$
7. $\frac{2\,a^2b^2}{2\,ab} =$
8. $\frac{3\,a^2c}{3\,a} =$
9. $\frac{4\,x^3y}{2\,xy} =$
10. $\frac{9\,cd^2}{3\,cd^2} =$
11. $\frac{15\,bc^4}{5\,bc} =$

301. NEGATIVE QUANTITIES IN DIVISION.

$$a^3 \div a = a^2; \quad -a^3 \div -a = a^2.$$

$$a^4 \div -a^2 = -a^2; \quad -a^3 \div a = -a^2.$$

In division, if both dividend and divisor are either positive or negative, the quotient is positive; if one is positive and the other negative, the quotient is negative.

WRITTEN EXERCISES

302. Divide:

1. $2x \div x$.
2. $3a \div 3$.
3. $-4y \div -26$.
4. $-6x \div -3$.
5. $14b \div 7$.
6. $18ax \div 9a$.
7. $8c \div -2c$.
8. $15x \div -3$.
9. $14axy \div -2xy$.

303. $3a + 6ab - 9a^2$ by $3a$.

$$3a)\overline{3a + 6ab - 9a^2}$$
$$1 + 2b - 3a.$$

If the dividend contains several terms, divide each separately and give each quotient its proper sign.

Divide: $a^2 + 2ab + b^2$ by $a + b$.

$$a + b)a^2 + 2ab + b^2(a + b$$
$$\underline{a^2 + ab}$$
$$ab + b^2$$
$$\underline{ab + b^2}$$

If each dividend and divisor has more than one term, divide as in long division in arithmetic, giving each term of the quotient its proper sign.

WRITTEN EXERCISES

304. Divide:

1. $a^3 + 3a^2b + 3ab^2 + b^3$ by $a + b$.

$$a + b)a^3 + 3a^2b + 3ab^2 + b^3(a^2 + 2ab + b^2$$
$$\underline{a^3 + a^2b}$$
$$2a^2b + 3ab^2$$
$$\underline{2a^2b + 2ab^2}$$
$$ab^2 + b^3$$
$$\underline{ab^2 + b^3}$$

2. $a^2 - b^2$ by $a + b$; by $a - b$.
3. $a^3 + 3a^2b + 3ab^2 + b^3$ by $a^2 + 2ab + b^2$.
4. $x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$ by $x + y$.
5. $4a^2 + 8ab + 4b^2$ by $2a + 2b$.

EQUATIONS

ORAL EXERCISES

305. Read the following equations:

1. $4+5=9$.
2. $3+4=2+5$.
3. $6-2=7-3$.
4. $8+1=12-3$.
5. $20-5=11+4$.
6. $6\times 2=3\times 4$.
7. $25\div 5=15\div 3$.
8. $\frac{16}{2}=\frac{24}{3}$.
9. $2\times 3=\frac{18}{3}$.
10. $\frac{48}{6}=2\times 4$.
11. $\frac{50}{5}=40\div 4$.
12. $30\div 5=2\times 3$.
13. $3\times 4=6+6$.
14. $8-2=3\times 2$.

306. In each of the above expressions the quantities separated by the sign $=$ are equal.

Each of the expressions is called an **equation**.

An equation indicates the equality of two quantities.

In equation (**2**) the sum of 3 and 4 = the sum of 2 and 5.

$3+4$ and $2+5$ are called **members** of the equation.

Name the members of the other equations.

Subtract 4 from each member of the second equation. The answer is $3=2+1$. Is this an equation?

Add 3 to each member of equation (**3**).

You have $6-2+3=7-3+3$. Is this an equation?

Subtract 5 from each member in equation (**4**). Perform the operations indicated. Have you still an equation?

Multiply each member of equation (**6**) by 2, thus:

$6\times 2\times 2=3\times 4\times 2$. Is the result an equation?

Divide each member of equation (**4**) by 3, thus:

$$\frac{8+1}{3}=\frac{12-3}{3}.$$

Perform the operations. Have you still an equation?

Does multiplying or dividing each member of an equation by the same number affect the equality?

WRITTEN EXERCISES

307. Perform the operations indicated in section 305 and see if the result in each case is an equation:

1. Add 3 to each member of equations (6), (7), and (8).
2. Subtract 4 from each member of equations (9) and (10).
3. Multiply by 2 each member of equations (11) and (14).
4. Divide by 3 each member of equations (12) and (13).

UNKNOWN QUANTITIES

308. In problems stated as equations the unknown quantity, that is, the number to be found, is often represented by one of the last letters of the alphabet, as x, y, or z.

Thus, $a + b + c = x$, letting $a = 2$, $b = 3$, and $c = 4$, would read $2 + 3 + 4 = x$.

Adding, we find $2 + 3 + 4 = 9$. Therefore in this example $x = 9$.

$2b - c = x$. That is, $2 \times 3 - 4 = x$.

Solving: We have $6 - 4 = 2$.

$x = 2$ in this example.

WRITTEN EXERCISES

309. Find the value of x in each of the following: if $a = 2$, $b = 3$, $c = 4$, $d = 5$:

1. $2a + 3b - c = x$.
2. $2d + c = x$.
3. $ab + cd - 2a = x$.
4. $\frac{c}{a} = x$.
5. $\frac{a + c}{b} = x$.
6. $\frac{ad + c}{c} = x$.
7. $\frac{3c}{b} + d = x$.
8. $\frac{2d - a}{c} = x$.
9. $\frac{4b + c}{a} - d = x$.

SOLVING EQUATIONS

310. Finding the Value of an Unknown Quantity.

Find the value of x in

$$x + 5 = 10.$$

A short method of subtracting 5 from both members is to move the 5 to the other member with its sign changed, thus:

$$x = 10 - 5 = 5.$$

Find the value of x in $\quad x - 3 = 6.$

Moving 3 to the other member with its sign changed, gives:

$$x = 6 + 3 = 9.$$

Moving a term from one member of an equation to the other, with its sign changed, is called **transposition.**

Transposing terms in this way does not affect the equality.

311. Find the value of x in the following equations:

$$4x - 10 = 3x + 6.$$

Transposing $3x$ from the second member of the equation to the first, we have

$$4x - 10 - 3x = 6, \text{ or } x - 10 = 6.$$

Transposing 10 to the second member, we have

$$x = 6 + 10. \qquad x = 16.$$

Both transpositions may be made at once, thus:

$$4x - 3x = 6 + 10. \qquad x = 16.$$

Proof: $x = 16$; $4x = 64$; $64 - 10 = 48 + 6$; $54 = 54$.

Finding the value of the unknown quantity in an equation is called **solving the equation.**

To solve an equation, first place the known quantities in one member and the unknown in the other. Then combine the unknown quantities into a single quantity and divide both members by its coefficient.

WRITTEN EXERCISES

312. Solve the following equations and state what you do to both members of the equation in each problem :

1. $4 + x = 16$.	**6.** $x + 20 = 45$.	**11.** $20 = 10 + x$.
2. $x - 5 = 18$.	**7.** $20 + x = 25$.	**12.** $35 = 20 + x$.
3. $x - 12 = 20$.	**8.** $x - 8 = 9$.	**13.** $18 + x = 28$.
4. $8 + x = 24$.	**9.** $14 + x = 36$.	**14.** $28 = 15 + x$.
5. $40 + x = 50$.	**10.** $x + 20 = 100$.	**15.** $50 = 20 + x$.

16. If $x = 8$, what is the value of $5x$?

17. If $x = 20$, what is the value of $3x$?

18. If $x = 10$, what is the value of $8x$?

How did you find the value of x in Examples 16, 17, and 18?

19. If $5x = 40$, what is the value of x?

20. If $3x = 60$, what is the value of x?

21. If $8x = 80$, what is the value of x?

313. If $\frac{1}{2}x = 5$, 2 times $\frac{1}{2}x = \frac{2}{2}x$, or $x = 2$ times $5 = 10$.

If $\frac{1}{3}x = 6$, what is the value of x?

If $\frac{1}{4}x = 3$, what is the value of x?

What has been done to both members of the equation to give an equation whose first member is x?

Both members of an equation may be multiplied or divided by the same number without destroying the equality.

Give and solve an equation in which the value of x may be found by addition.

Give and solve an equation in which the value of x may be found by subtraction.

Give and solve an equation in which the value of x may be found by multiplication.

Give and solve an equation in which the value of x may be found by division.

WRITTEN EXERCISES

314. Solve the following equations:

1. $20 + x = 41$.
2. $x - 30 = -6$.
3. $x + 15 = 12$.
4. $x + 7 = -8$.
5. $20 - x = -10$.
6. $2x + x = 30$.
7. $3x - 10 = 2$.
8. $4x + 12 = 4$.
9. $5x - 15 = 5$.
10. $14 - x = 13$.

Transpose x when, in the final result, it would be negative if not transposed.

11. $4x - 21 + 3x = ?$
12. $10x + 16 - 7x = ?$
13. $16x - 4 - 6x + 8 = 34$.
14. $4x - 16 = 2x + 16$.
15. $15x - 15 - 3x + 5 = 4x + 6$.
16. $20 - 3x + 15 - 2x = 6 - 4x$.
17. $7 + 7x - 8 - 8x = -2$.
18. $2 + 3 - 18 - 30 = -11x$.
19. $x + 3 - 30x + 40 - 6x - 5 = 3x$.
20. $20 + 1 - 6x + 2 - 7x - 3 = 13 + 6x$.

SOLUTION OF PROBLEMS BY EQUATIONS

315. How many dollars added to \$10 will give \$15?

Let x = the number of dollars that added to \$10 will give \$15. Then $\$10 + x = \$15$. $\$10 + x = \$15$ is an equation.

What is the first member of this equation? The second?

If 10 is subtracted from each member of the equation, will the equality be preserved? What will the equation be?

What have we found the value of x to be?

If 16 be added to a number, it will equal 85. What is the number?

Let x = the number.

Then $x + 16 = 85$.

Subtracting 16 from both numbers; that is, transposing 16, $x = 69$.

WRITTEN EXERCISES

316. Solve:

1. Henry bought 27 marbles. He then had 100 marbles. How many had he at first?

2. After losing 30 marbles Fred had 115 left. How many had he at first?

3. The number of telephone calls allowed for a certain amount a year is 600. 375 have been made. How many calls are still due?

4. 9 times the number of girls in a class is 288. How many girls are in the class?

5. Jane was hunting eggs. Had she found 35 more she would have had 12 dozen. How many did she find?

6. A fox pursued by a hound has run 310 rods, which is 42 rods more than the hound has run. How far has the hound run?

7. 4 times the distance around a certain block in a city is 64 rods. What is the distance around the block?

8. 8 times the distance around a race track is 280 rods. How far is it around the track?

9. If 3 times a certain number be added to it, the sum will equal 36. What is the number?

10. After buying $\frac{1}{2}$ as much wheat as he raised, Mr. Huyer has 450 bushels. How many bushels did he raise?

11. Five times a number, less 40, equals three times the number, plus 20. What is the number?

12. If one half of a number be added to itself, the sum will be 48. What is the number?

13. Three times Mr. Holbrook's money, less 10 dollars, equals his money plus 25 dollars. How much money has he?

14. A peach grower bought of one man as many peaches as he had of his own; of another one half as many. He then had 2,000 baskets. How many did he buy?

15. Six times the distance run by some boys in a race, less 60 rods, was 4 times the distance, plus 30 rods. How far did they run?

16. After selling 50 sheep, a farmer had three fifths of his flock left. How many sheep had he?

17. Luther said, "If I had 60 cents more than I have, I should have three times as much as I now have." How much has he?

18. Amos said if he had twice as much money and one half as much as he now had, he would have $500. How much has he?

19. A farmer raised a certain number of bushels of rye, twice as many bushels of oats, and three times as many bushels of wheat. He had 1,200 bushels in all. How many bushels of each had he?

20. Horace has 15 cents more than Charles. If each is given 5 cents, Horace will then have twice as much as Charles. How many cents have each?

21. A mason and a plumber together earn $9 a day. The mason works 18 days and the plumber 23 days. They earn together $182. What does each earn per day?

22. Mrs. Lynn paid 72 cts. for flour and sugar, buying the same number of pounds of each. For the flour she paid 3 cts. per pound and for the sugar 6 cts. per pound. How many pounds of each did she buy?

23. Mr. Leonard's wages were $4 per day and his son's wages were $2 per day. Both worked the same number of

days during March. The amount of their earnings was \$144. How many days did each work?

24. The earnings of Mr. Danforth and his son amounted to \$180. Mr. Danforth worked three times as many days as his son. Mr. Danforth's wages were \$5 per day and his son's \$3 per day. How many days did each work?

317. FRACTIONAL EQUATIONS.

Find the value of x in the equation $\frac{x}{2} = 4$.

$\frac{x}{2}$ is the same as $\frac{1}{2}$ of x, which equals $\frac{1}{2} \times x$, or $\frac{1}{2}x$.

That is, in $\frac{x}{2}$, the coefficient of x is $\frac{1}{2}$.

The equation, then, is $\frac{1}{2}x = 4$.

(1) When x, having a coefficient, stands alone as one member of an equation, its value is found by dividing both members by the coefficient.

In $\frac{1}{2}x = 4$, divide both numbers by $\frac{1}{2}$.

We have $(\frac{1}{2} \div \frac{1}{2})x = 4 \div \frac{1}{2}$; or,

$(\frac{1}{2} \times \frac{2}{1})x = 4 \times \frac{2}{1}$.

$x = 8$.

(2) A shorter way to find the value of x is to multiply each member of the equation by the denominator. Thus (remembering that $\frac{1}{2}x = \frac{x}{2}$):

$$\frac{x}{2} = 4.$$

$$\frac{x}{2} \times 2 = 4 \times 2; \quad \frac{2x}{2} = 8.$$

$$x = 8.$$

(3) If there is a fraction in both members find a common denominator, and multiply both members by it.

Find the value of x in the equation $\frac{x}{2}=\frac{2}{3}$.

Here both members are fractions.

$$\begin{array}{l} 3 \quad 4 \\ \frac{\not{6}\,x}{\not{2}}=\frac{\not{12}}{\not{3}} \\ 3x=4. \\ x=\frac{4}{3}=1\frac{1}{3}. \end{array}$$

Removing the fractions from an equation is called **clearing the equation of fractions.**

Make a rule for clearing an equation of fractions.

WRITTEN EXERCISES

318. Solve:

1. $\frac{x}{2}=30.$
2. $\frac{x}{3}=20-2x.$
3. $\frac{2x}{4}=15.$
4. $\frac{3x}{5}=24-x.$
5. $\frac{5x}{7}=5-x.$
6. $\frac{x}{4}=\frac{6}{12}.$
7. $\frac{x+3}{5}=\frac{4}{5}.$
8. $\frac{2x-2}{4}=\frac{1}{2}.$
9. $\frac{3x-6}{8}=\frac{3}{4}.$
10. $\frac{2-x}{16}=\frac{1}{16}.$
11. $\frac{x+2}{3}=\frac{x-5}{2}.$
12. $\frac{2x-1}{4}=\frac{3x-2}{4}.$
13. $\frac{1+5x}{6}=\frac{3+x}{2}.$
14. $\frac{4x-6}{2}=\frac{2x+9}{5}.$
15. $\frac{5x+3}{7}=\frac{6x+6}{9}.$
16. $\frac{1}{x+2}=\frac{2}{6}.$
17. $\frac{3}{2x-1}=\frac{4}{x+2}.$
18. $\frac{4}{5x+8}=\frac{5}{7x+7}.$

319. SOLUTION OF PROBLEMS INVOLVING FRACTIONAL EQUATIONS.

The sum of two numbers is 320, and the quotient of the greater number divided by the less is 3. What are the numbers? *Ans.* 240 and 80.

SUGGESTION: $x =$ smaller.
$320 - x =$ larger.

WRITTEN EXERCISES

320. Solve:

1. Mr. Reed paid for his carriage $\frac{5}{8}$ as much as he paid for his horse. The horse cost \$64 more than the carriage. What was the cost of each?

2. Five times Anne's money increased by $\frac{4}{5}$ of her money is equal to \$464. How much money has she?

3. The difference between $\frac{3}{4}$ of a man's age and $\frac{2}{3}$ of his age is 4 years. How old is he?

4. Mr. Hudson, Mr. Childs, and Mr. Lewis engaged in business with a capital of \$14,000. Mr. Childs put $\frac{1}{2}$ as much money in the business as Mr. Hudson, and Mr. Lewis put in $\frac{1}{2}$ as much as Mr. Childs. What part of the capital did each provide?

HINT: $x =$ Mr. Lewis's money.

5. The weight of a team with a load of coal is 5500 lb. The team weighs $\frac{4}{5}$ as much as the coal, and the wagon weighs $\frac{1}{2}$ as much as the team. What is the weight of each?

6. Harry and his sister Celia have together \$.64. Celia has 12 cts. more than Harry. How much have each.

7. Mr. Kennedy sold two plots of land containing 80 acres in all. One lot is 10 acres larger than the other. What is the size of each plot?

8. In a school of 1,200 pupils the girls outnumbered the boys by 120. How many were there of each?

9. Mr. Harris spent $2 for 5-cent stamps, 2-cent stamps and postal cards. He bought 30 more 2-cent stamps than 5-cent stamps, and 30 more postal cards than 2-cent stamps. How many did he buy of each?

Let x = number of 5-cent stamps.
$x + 30$ = number of 2-cent stamps.
$x + 60$ = number of postal cards.

Then $5x + 2(x + 30) + x + 60 = \$2.$

10. The sum of two numbers is 30. Three times the first number added to four times the second is 104. What are the numbers?

x = the first number.
$30 - x$ = the second number.
$3x + 4(30 - x) = 104.$

11. There are 865 pupils in a school. The girls exceed the boys by 15. What is the number of each?

12. $2,400 is divided among three children. The second receives twice as much as the first, and the third receives five times as much as the first. What is the share of each?

13. 175 is the difference between six times a certain number and eleven times the same number. What is the number?

14. The sum of three numbers is 240. The second is double the first and the third is four times the sum of the other two. What are the numbers?

15. Frank and Edward had together 72 marbles. Edward gave Frank four of his marbles. Frank then had twice as many as Edward. How many marbles had each?

16. The sum of the ages of a father and a son is 68 years. In 6 years the father's age will be three times the age of the son. What is the age of each?

17. On Saturday, Sept. 14, 1907, Mr. Dalrymple deposited 45 bills in a bank. Some of them were $5 bills and the others $10 bills. The amount deposited was $325. How many of each did he deposit?

18. Mr. Macy sold 4 baskets of apples and 8 baskets of peaches for $13.80, and 8 baskets of apples and 4 baskets of peaches for $9.60. How much did he receive for a basket of apples? Of peaches?

19. The ages of Mr. Matthews and his son together make 98 years. Ten years ago Mr. Matthews was twice as old as his son. What is the age of each now?

SUMMARY OF CHAPTER VI

CHAPTER VII

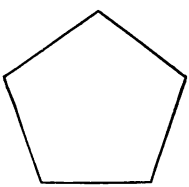
MENSURATION

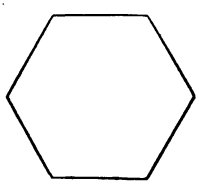
PLANE FIGURES

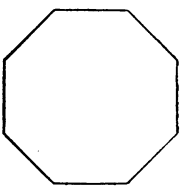
POLYGONS

321. What is a plane figure? Name five plane figures.

A plane figure bounded by straight lines is called a **polygon**. Squares, triangles, hexagons, are examples of polygons.





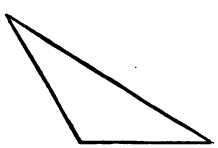


A polygon is named from the number of its sides or angles.

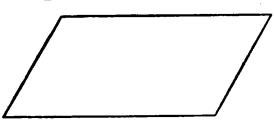
How many sides has a triangle? A quadrangle? A pentagon? A hexagon? An octagon? How many angles has each figure named?

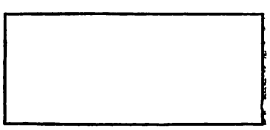
In previous lessons, the following rules for finding the areas of plane figures have been learned:

1. The area of a triangle is equal to $\frac{1}{2}$ the product of the base by the altitude.

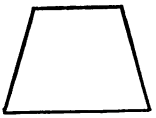


2. The area of a rectangle is equal to the product of the length by the breadth.





3. The area of a parallelogram is equal to the product of the base by the altitude.

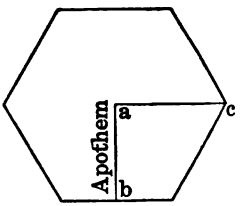


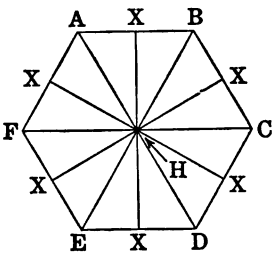
4. The area of a trapezoid is equal to $\frac{1}{2}$ the sum of the two parallel sides multiplied by the altitude.

A polygon having all its sides equal and all its angles equal is called a **regular polygon**

Equilateral triangles and squares are examples of regular polygons.

A line drawn from the center of a regular polygon, perpendicular to any side, is called its **apothem**. The line AB is the apothem of this hexagon. The line AC is the radius of the hexagon. Make a definition for the radius.





322. Into how many equal triangles do the radii of the hexagon divide it?

The apothem is the altitude of each triangle. How do you find the area of each triangle?

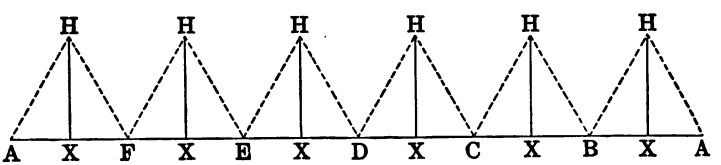
Area of the triangle AHB = AB $\times \frac{1}{2}$ HX.

Area of the hexagon

= (AB + BC + CD + DE + EF + FA) $\times \frac{1}{2}$ HX

= the perimeter, ABCDEF $\times \frac{1}{2}$ HX, or

$\frac{1}{2}$ of ABCDEF $\times$ HX.



The area of a regular polygon = $\frac{1}{2}$ the product of its perimeter by its apothem.

WRITTEN EXERCISES

323. Solve:

1. Find the area of a regular hexagon 6 ft. on a side, its apothem being 5.2 ft.

2. An eight-sided fountain is 9 ft. on a side. The perpendicular distance from the center to the middle of each side is 10.86 ft. What is the area of the fountain?

3. A barn is 40 feet wide. The distance from the peak to the beams or plate is 14 feet. What is the area of the gable end? Draw a figure showing the gable end.

4. A department store 60 x 40 feet has four floors. What is the total floor area?

5. A piece of land in the form of a parallelogram contains 2 acres. It is 20 rods between the two parallel sides. What is the length of the other two sides?

6. A tapering board is 12 feet long. It is 14 inches wide at one end and 8 inches at the other. How many square feet does it contain?

7. The new park is in the shape of a trapezoid. One of the parallel sides is $\frac{1}{2}$ mile long, the other 120 rods. It is 85 rods in width. How many acres does it contain?

8. The cellar door is $5 \times 3\frac{1}{2}$ ft. How many square feet of boards in it?

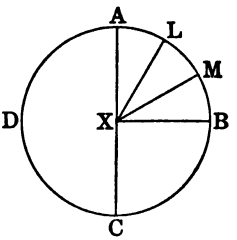
9. It slopes from the house, the upper end being 3 ft. above the ground. How many square feet in the triangular supports on each side, the base lines being 4 ft.?

10. Draw diagrams of the top and the supports on the scale of $\frac{1}{2}$ in. to 1 ft.

11. The pages of a book are $7\frac{1}{2}$ in. $\times$ 5 in. There is a margin of 1 in. at top and bottom and of $\frac{3}{4}$ in. on each side. What is the length of a line of print? A page?

MEASUREMENT OF ANGLES

324. The circumference of every circle, large or small, is supposed to be divided into 360 equal parts, called **degrees** (°); each degree into 60 **minutes** (′); and each minute into 60 **seconds** (″).



Thus: the circumference of the circle ABCD = 360°.

A half of the circumference AC = 180°.

A quarter of the circumference AB = 90°.

What kind of angle is AXB? What part of the circumference measures it? How many degrees is it? How many degrees in a right angle?

All angles are measured in degrees, minutes, and seconds, by the parts of the circumference opposite them, the center being at the apex of the angle.

A part of the circumference of a circle is called an **arc.**

Thus, the angle AXB is an angle of 90° and is measured by the arc B.

The angle AXM is an angle of 60°. Name its arc.

The angle AXL is an angle of 30°.

TABLE

60 seconds = 1 minute; 60″ = 1′
60 minutes = 1 degree; 60′ = 1°
360 degrees = 1 circle
4 right angles, 360° = 1 circle

What are degrees of longitude?

Where is the apex of the angle that is measured by a degree of longitude?

CIRCLES

325. A circle is a plane figure bounded by a curved line, every point of which is equally distant from a point within called the center.

Draw a circle. What is its circumference? Draw a radius; a diameter.

What is the ratio of the circumference of a circle to its diameter?

Explain the formula $c = 2\pi r$.

What does π equal?

1. What is the circumference of a circle whose diameter is 6 in.?

SOLUTION: $c = d \times \pi$.

3.1416×6 in. $= 24.8496$ in., circumference.

2. What is the circumference of a circle whose radius is 2.5 in.?

SOLUTION: $c = 2r \times \pi$.

$2 \times 2.5 \times 3.1416 = ?$

What is the radius of a circle whose circumference is 15.708 in.?

SOLUTION: $c \div \pi = 2r$.

$15.708 \div 3.1416 = 5 = 2r$.

$5 \div 2 = 2\frac{1}{2} = r$.

WRITTEN EXERCISES

326. Solve:

	GIVEN		FIND
1.	$c = 47.124$	ft.	d.
2.	$c = 78.53$	in.	r.
3.	$r = 12$	in.	c.
4.	$d = 16$	ft.	c.
5.	$r = 10.5$	in.	c.

6. The bottom of my ink bottle measures 3 in. across. What is its circumference?

7. The minute spaces on the tower clock face are 1 ft. apart. What is the diameter of the face?

8. What is the diameter of a circular one-mile race track? Of a water main 3 ft. in circumference?

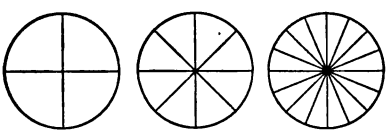
9. Miss Jewett had a new circle painted on the floor of the kindergarten, large enough to provide room for 25 children, allowing 2 ft. for each child. What was the diameter of the circle?

10. Ten times around the track in Madison Square Garden, New York, make 1 mile. What is the diameter of the track?

11. A smokestack $3\frac{1}{2}$ ft. in diameter is made of sheet iron. How wide must the sheets be cut to make it, allowing 10 in. for lapping?

AREA OF A CIRCLE

327. Draw a circle having a radius of 3 in. and divide it into 16 equal parts by drawing 8 diameters at right angles, thus: —



Each section may be divided into still smaller sections.

It is plain that the circumference may be thought of as unrolled into a straight line, and hence as made up of any number of very short straight lines, which may be arranged in any form of polygon.

Hence a circle may be thought of as a regular polygon having a great number of very short sides, with the radius for its apothem.

How do you find the area of a regular polygon?

Area of a regular polygon $= \frac{1}{2}$ perimeter $\times$ apothem.

Hence, *area of a circle* $= \frac{1}{2}$ *circumference* $\times$ *radius*.

Circumference $= 2\pi \times r$.

Area $= \frac{1}{2}\, 2\pi \times r \times r = \frac{1}{2}\, 2\pi \times r^2 = \pi \times r^2$, or πr^2.

The area of a circle is found by multiplying the square of the radius by 3.1416.

Find the area of a circle of $3''$ radius.

$3''^2 = 9$ sq. in.

9 sq. in. $\times$ 3.1416 = 28.2744 sq. in., area.

WRITTEN EXERCISES

328. Solve:

1. Find the area of a circle whose radius is 4 in.

2. Find the area of a circle whose diameter is 10 in.

3. How many square feet of boards are required to floor a circular summer house whose diameter is 16 ft.?

4. How many square feet are there in the surface of a circular pond having a circumference of 6,854 ft.?

5. How many square feet in the surface of the stump of a redwood whose diameter is 4 ft. 6 in.?

6. How many square feet in a right triangle whose base is 17 in. and altitude 21 in.?

7. What is the area of a cross section of a 1-inch pipe?

8. What is the area of a cross section of a 2-inch pipe? Note that the area of the cross section of the 2-inch pipe is 4 times as great as that of the 1-inch pipe.

9. What is the area of a cross section of a 3-inch pipe? This is 9 times as great as the area of a cross section of the 1-inch pipe. How many times as great would a 4-inch pipe be? A 5-inch pipe?

MEASUREMENT OF SOLIDS

329. A **solid** is a figure having 3 *dimensions*, — length, breadth, and thickness.

The space occupied by a solid is called its **solid contents** or **volume.**

Parallelopipeds. A solid having all its edges straight and all its sides parallelograms is called a **parallelopiped.**

A square box, the schoolroom, a book, are **parallelopipeds.**

The solid contents of a parallelopiped are found by multiplying together its three dimensions.

PRISMS

330. A prism is a solid whose sides are parallelograms and whose two ends, or bases, are equal polygons.

A prism is named from the shape of its bases, as, *triangular*, *square*, *hexagonal*, *octagonal.*

The altitude of a prism is the perpendicular distance between the bases.

A prism whose altitude is at right angles to its bases is called a **right prism.**

The total area of the sides is called the **convex surface.**

The area of the convex surface plus the area of the bases is the total area.

WRITTEN EXERCISES

331. Solve:

1. How many cubic feet in a stick of timber 40 ft. long, 9 in. wide, and 5 in. thick?

2. Name some objects that are good examples of regular parallelopipeds.

3. Find the cubical contents of one.

Area of the Surface of a Prism.

How many square inches in the convex surface of the prism represented by this cut which is drawn on the scale of 1 in. to 6 in.?

How many square inches in each side? In all the sides?

$$3 \times 3 \times 2 = 9 \times 2 = 18. \quad \textit{Ans.}\ 18 \text{ sq. in.}$$

How many square inches in the ends?

How do you find the area of a triangle?

The area of each end $= \frac{1}{2}$ base $\times$ apothem $= \frac{1}{2} 3 \times 2.6 = 3.9$.

The area of the surface of the prism $= 18$ sq. in. $+ 3.9$ sq. in. $+ 3.9$ sq. in. $= 25.8$ sq. in.

Area of surface of a prism = perimeter of base $\times$ altitude $\times$ base $\times$ apothem.

1. Find the area of the surface of a prism 6 ft. high with a base of 2 ft. square.

2. Find the area of the surface of a hexagonal monument in the shape of a prism 10 ft. high and 3 ft. on a side. The apothem of the base is 4.12 ft.

CYLINDERS

332. What is a circle?

Define circumference, radius, diameter.

Explain the formula, circumference $= 2\pi r$.

A **cylinder** is a solid having a uniformly curved surface and its ends similar and equal.

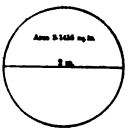
The **altitude** of a cylinder is the distance between the bases.

In regular cylinders the altitude is at right angles to the base.

NOTE. The pupils should each be furnished with a cylinder 2 in. in diameter and 4 in. high; a circular box will answer the purpose.

333. To find the Area of the Surface of a Cylinder.

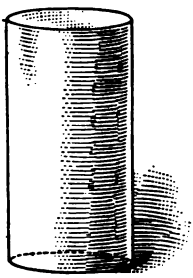
From a sheet of paper cut circles to fit the top and bottom of the cylinder. Then fit a piece of paper around the curved surface. Mark it and cut it out.



To find the area of each end.

How do you find the area of a circle?

$$\text{Area} = \pi r^2.$$



The diameter of the cylinder given is 2 in.

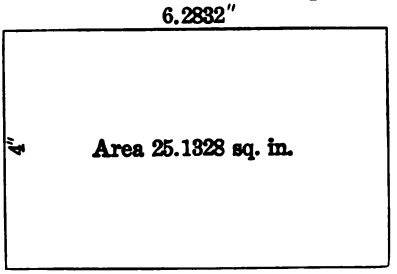
$r = 1$ in. $r^2 = 1$.

The area $= 3.1416 \times 1 = 3.1416$.

To find the area of the curved surface.

Spread out the paper that fits around the curved surface of the cylinder. What is its shape?

Its length corresponds with the circumference of the cylinder. Mark the length on the rectangle.



With what does its width correspond? Mark it on the rectangle.

How do you find the area of a rectangle?

Area of the curved surface of a cylinder = the circumference × the altitude.

Find the entire area.

3.1416 sq. in. in top.
3.1416 sq. in. in bottom.
25.1328 sq. in. in curved surface.
31.4160 sq. in. in surface of cylinder.

Suggestion to Teacher. Hold up a large cylinder and give dimensions, as diameter 4″, altitude 8″. Let the pupils draw to scale.

334. To find the area of the surface of the cylinder.

(1) Find the areas of the ends:

$$\pi r^2 = 2^2 \times 3.1416.$$
$$4 \times 3.1416 = 12.5664.$$
$$12.5664 = \text{sq. in. in top.}$$
$$12.5664 = \text{sq. in. in bottom.}$$

(2) Find the area of the curved surface:

$$12.5664 \text{ in. in circumference.}$$
$$\underline{\qquad 8 \text{ (height)}}$$
$$100.5312 \text{ sq. in. in curved surface.}$$

(3) Find the entire area:

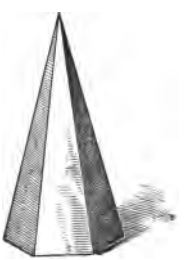
$$12.5664 \text{ sq. in. in top.}$$
$$12.5664 \text{ sq. in. in bottom.}$$
$$\underline{100.5312 \text{ sq. in. in curved surface.}}$$
$$125.6640 \text{ sq. in. in surface of cylinder.}$$

1. Take the necessary measures of some cylinder. Draw a diagram of it to scale.

2. Find the areas of the ends; of the curved surface; the area of the entire surface.

THE PYRAMID

335. *A pyramid* is a solid having a polygon for its base and its surface made of triangles meeting at a point, called the **apex**.



The triangles making the surface of a *regular pyramid* are all *isosceles*.

The perpendicular distance from the apex of the pyramid to the base is called its **altitude**. The altitude of one of its triangles is called the **slant height** of the pyramid.

336. To find the Area of the Surface of a Pyramid.

How do you find the area of an isosceles triangle?

How, then, can you find the area of the surface of a pyramid?

The area of the surface of a regular pyramid = the perimeter of its base × ½ its slant height.

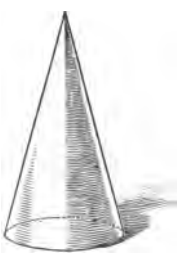
1. How many square feet in the surface of a square pyramid each of whose sides has a base of 3 ft. and a slant height of 6 ft.?

Solution: Perimeter × ½ slant height = 4 × 3 × 3 = 36.

2. How many square feet are there in the surface of a pyramid, the base of which has 6 edges, each 30 ft. long, and whose slant height is 20 ft.?

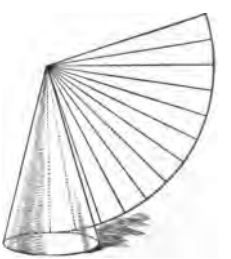
3. A pyramid with a square base is 20 ft. on a side. The slant height is 12 ft. The pyramid is to be covered with paper at $.05 a square foot. How much will it cost?

THE CONE



337. *A cone* is a solid having a curved base and tapering evenly to a point called the *apex* or *vertex*.

Most cones have circles for bases and are called *circular cones*.



As the circumference of a circle may be thought of as made up of a very great number of very short straight lines, so the surface of a cone may be thought of as made of a very large number of very narrow isosceles triangles. Hence:

The area of the surface of a cone = the perimeter of its base by ½ its slant height.

WRITTEN EXERCISES

338. Solve:

1. Find the area of the surface of a cylinder having a diameter of 6 inches and an altitude of 8 inches. Represent by drawing.

2. Helen has a cylindrical box 8 inches in diameter and 5 inches high. She wishes to line this with silk and give it to her brother for a collar box on his birthday. How many square inches of silk will she need for the lining if she adds 60 extra square inches for turning in?

3. How many square feet of galvanized iron will it take to make a boiler $2\frac{1}{2}$ feet in diameter and 7 feet long?

4. The Pennsylvania Railroad Company erected a water tank 12 feet in diameter and 22 feet high. How many square feet of sheet iron were required to line it?

5. How many square feet of sheathing does it take to cover a six-sided steeple 5 feet on a side at the base, having a slant height of 75 feet?

6. The fountain in the park is hexagonal, 8 feet on a side. In the winter it is covered with a board cover in the shape of a pyramid whose slant height is 16 feet. How many feet of boards does it contain?

7. Find the area of the surface of a square pyramid, 10 ft. $\times$ 10 ft. at the base, and having a slant height of 18 feet.

8. At 32 cts. a square yard what will it cost to paint a round steeple whose slant height is 80 feet, and its diameter 8 feet at the base?

9. How much cardboard will be required to make a cornucopia 3 inches in diameter at the mouth and 10 inches deep, measured on the outside?

THE SPHERE

339. A **sphere** is a solid bounded by a curved surface, every point of which is equally distant from a point within called the center.

The **diameter** of a sphere is any straight line passing through the center and ending at opposite points of the surface.

The **radius** of a sphere is any straight line from the center to the surface.

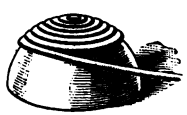
A section of a sphere passing through the center is called **a great circle** of the sphere.

Every great circle of a sphere divides it into two equal parts, or **hemispheres.**

The **circumference** of a sphere is the circumference of one of its great circles.

340. TO FIND THE AREA OF THE SURFACE OF A SPHERE.

NOTE. If possible, get one or more models of hemispheres and wind the curved and flat surfaces, as shown in the picture. Then measure the cords used. It is well to wax the cords.



The length of the cord used to cover the curved surface of a hemisphere, as shown in the picture, is exactly twice that used to cover the flat surface. Hence:

The area of the curved surface of a sphere is 4 times the area of its great circle.

How do you find the area of a circle?

The area of a circle $= \pi r^2$.

The curved surface of a sphere $= 4 \pi r^2$.

Find the convex surface of a ball 4 in. in diameter.

SOLUTION: $r^2 = 2^2 = 4$

$\pi = 3.1416$

$4 \times 4 \times 3.1416 = Ans.$

WRITTEN EXERCISES

341. Solve:

1. How much leather will it take to cover a push ball 8 ft. in diameter?

2. The gilded ball on the flagpole of one of the high buildings in Seattle in $2\frac{1}{2}$ ft. in diameter. How much will it cost to gild it at $12\frac{1}{2}$ cts. a square foot?

3. How many square feet in the surface of an 18-in. globe?

4. What is the convex surface of a cannon ball 6 in. in diameter?

5. How many square inches of leather does it take to cover a baseball $3\frac{1}{2}$ in. in diameter?

VOLUMES OF SOLIDS

342. Lay cubical 1-inch blocks to form a parallelopiped $3 \times 3 \times 2$ in. How many cubic inches does it contain?

To find the volume of a parallelopiped, multiply together its three dimensions.

WRITTEN EXERCISES

343. Solve:

1. Find the cubic contents of a parallelopiped 6 inches long, 4 inches wide, and 3 inches high.

2. How many cubic inches in a 4-inch cube? In a 5-inch cube? In a 6-inch cube? In an 8-inch cube?

3. What is the ratio of a 6-inch cube to a 3-inch cube?

4. What is the ratio of an 8-inch cube to a 4-inch cube?

VOLUME OF A PRISM

344. Lay four 4-inch cubes in a square. How many cubic inches in the square? $4 \times 1 = 4$.

Lay another row. How many cubic inches have you now? $4 \times 2 = 8$.

Lay three more rows. How many cubic inches have you now? $5 \times 4 = 20$.

What kind of a solid have you made?

How do you find the volume of a regular prism?

Area of base × altitude = volume.

WRITTEN EXERCISES

345. Solve:

1. A certain pillar is 8 ft. high and 2 ft. square. What is its volume?

2. An office building is 2,150 ft. high and 80×40 ft. on the ground. What are its solid contents?

3. I desire to build a triangular corner cupboard, 6 ft. high, extending 3 ft. along each wall. What will be its solid contents? Draw a diagram.

SUGGESTION: How do you find the area of a right triangle?

4. The hexagonal base of the Soldiers' Monument is 6 ft. high. Its sides are each 9 ft. and its apothem is 10.86 ft. How many cubic feet of stone were required for its construction?

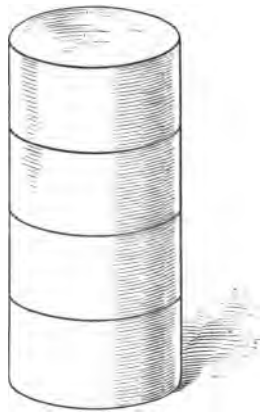
5. Is a brick an example of a square prism? Is a foundation wall?

6. How many bricks 8 in. × 4 in. × 2 in. will be required for a solid wall 10 ft. long, 8 ft. wide, and 8 ft. high?

7. If a person requires 50 cu. in. of air per minute, how long will the air in a schoolroom 30 ft. by 24 ft. by 14 ft. last? Would this volume of air be a prism?

VOLUME OF A CYLINDER

346. The figure represents a cylinder whose base has an area of 4 square inches. How many cubic inches in one layer 1 inch high?



If the cylinder is 4 inches high, how many cubic inches does it contain?

4 cu. in. in 1 layer.
4 (layers)
16 cu. in. in the cylinder.

Volume of a cylinder = area of the base × height (h) = $\pi r^2 \times h$.

Find the volume of a cylinder 2 in. in diameter and 4 in. high.

πr^2 (the area of the base) = 3.1416 sq. in.

4 in. (height) × 3.1416 = 12.5664 cu. in., solid contents, or volume.

Find the volume of a cylinder having a diameter of 8 in. and altitude of 12 in.

SUGGESTION: $\pi r^2 = 3.1416 \times 16 = 50.2656 =$ sq. in. in base.

$\pi r^2 \times h = ?$

1. A cylinder is 4 in. in diameter and 12 in. high. What is its volume?

2. If its diameter and its height were both doubled, what would the volume be?

3. Which of the following are cylinders: a cheese, stove pipe, a silver dollar, a lead pencil, a piece of pipe?

4. Take the necessary measurements of some cylinder in your schoolroom. Draw a diagram of it to scale, and find its volume.

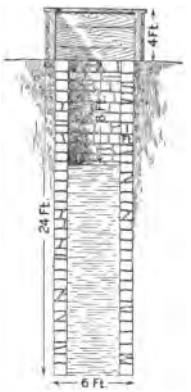
WRITTEN EXERCISES

347. Solve:

1. Find the number of cubic feet of earth that will be removed in digging a cistern 4 feet in diameter and 18 feet deep.

2. A circular pond was 2,500 feet in diameter, and had an average depth of 5.5 feet. How many cubic feet of water did it contain?

3. How many cubic yards of earth will be removed in digging a circular pit 12 feet in diameter and 3 feet deep?



4. In a church there were 12 stone pillars, each 18 feet high and 7.854 feet in circumference. How many cubic feet of stone in all the pillars?

5. How many cubic feet of water in a cylindrical boiler 4 feet in diameter and $6\frac{1}{2}$ feet long?

6. Mr. Westcott dug a well 24 ft. deep and 6 ft. in diameter. How many cubic feet were excavated?

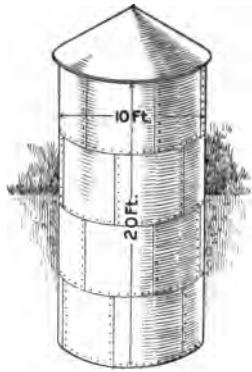
7. He lined it with a stone wall 1 ft. thick. What was the diameter of the well then?

8. When water stands in it 8 ft. from the top, how many gallons does it contain?

9. A square curb is built over the well, 4 ft. high and just reaching to the outside of the stone lining. How many feet of boards are required for the curb?

10. Mr. Horton built a silo* near his barn. It was 10 ft. in diameter and 20 ft. high. How many square feet of lumber were required to cover the sides?

*NOTE. A silo is a cylindrical building used to hold certain kinds of feed for cattle, called *ensilage*.



11. It had a flat tin roof. How many square feet of tin were required to cover it?

12. $\frac{1}{2}$ of the silo was below the surface of the ground. How many cubic feet of earth were excavated for it?

13. How many bushels of ensilage will the silo hold?

A bushel contains 2150.42 cu. in.

14. How many cubic feet of gas will a pipe 6 in. in diameter and 1 mile long hold?

15. The new gas house is 25 ft. in diameter and 40 ft. high when fully distended. How many cubic feet of gas will it contain?

16. Mary's mother made a cake for her on her tenth birthday. Its diameter was 10 inches and its height 3 inches. What was its circumference? How many square inches in its top? How many cubic inches of cake were there in all?

17. If the cake was cut into eight equal pieces, what was the length of the arc of a circle represented by the rounded part of each piece?

18. How many degrees in the acute angle of each piece?

CAPACITY IN GALLONS *

348. How many cubic inches are there in a gallon? How many cubic inches are there in a cubic foot?

* Represent 1 gallon and 1 cubic foot, either by having the actual measures, or something about the same size, as a waste basket, hat box, can, etc.

$\frac{7\frac{111}{231}\text{ times}}{231)1728\text{ cu. in.}} = \frac{111}{231}$, nearly $\frac{1}{2}$. There are approximately $7\frac{1}{2}$ gallons in 1 cubic foot. In problems use $7\frac{1}{2}$.

1. How many gallons in 2 cubic feet?
2. How many gallons in 10 cubic feet?
3. How many gallons in 20 cubic feet?

How many gallons of water will a tank hold that is 5 feet in diameter and 6 feet deep?

$$\text{area} = \pi r^2.$$

$$\tfrac{1}{2} \text{ of } 5 = 2.5, \text{ radius.}$$

$$2.5 \times 2.5 = 6.25 = \text{radius squared.}$$

$$3.1416 \times 6.25 = 19.635000 \text{ sq. ft., area of base.}$$

$$\begin{array}{r} 19.635 \text{ sq. ft in base.} \\ 6 \text{ (altitude)} \\ \hline 117.810 \text{ cu. ft. in tank.} \end{array}$$

$$7.5 \times 117.81 = 883.575 \text{ gallons of water in the tank.}$$

WRITTEN EXERCISES

349. Solve:

1. How many gallons of water can be let into a cylindrical swimming tank 40 feet in diameter and 8 feet deep? Into a tank $36\frac{1}{2}$ feet in diameter and 9 feet deep?

2. How many gallons of water will be contained by a round boiler that is $3\frac{1}{2}$ feet in diameter and 8 feet high? By one that is 5.5 feet in diameter and 12.4 feet high?

3. How many gallons of oil will the cylindrical tank on an oil wagon hold, if it is 43.9824 feet in circumference and 15 feet long?

4. How many hours will it take to empty a cylindrical tank whose diameter is 10 feet and altitude 12 feet, if the pipe that carries off the water empties 12 gallons per minute?

5. How many gallons of water will a cylindrical cistern contain, whose diameter is 3 feet and depth 15 feet?

6. How many gallons of paint will a cylindrical vat contain that is $6\frac{1}{2}$ feet in diameter and 4 feet deep?

7. The Central Railroad of New Jersey erected a cylindrical water tank to supply their trains. This tank was 12 feet in diameter and 20 feet high. How many gallons of water would it hold?

8. How many gallons of water can be held at one time in a hose whose diameter is 2 inches and length 500 feet? (Use 231 inches to a gallon.)

9. In a factory there are six boilers, each 5 feet in diameter and 9 feet long. How many gallons of water will all six contain?

10. There are $31\frac{1}{2}$ gallons in a barrel. How many barrels will 63 gallons fill? 126 gallons? 315 gallons?

11. How many barrels can be filled with oil from a tank 14 feet in diameter and 7 feet deep?

12. The Northern Pacific Railroad erected a water tower for supplying passing trains. This tower was 47.124 feet in circumference and 22 feet high. How many square feet of sheet iron were required to line it?

13. How many gallons of water will this tower hold?

14. How many cubic feet of ice can be cut from a circular pond 120 feet in diameter, if the ice is frozen to the depth of $2\frac{1}{2}$ feet?

15. A New Orleans mill had a vat 12 feet in diameter and 4 feet deep. How many barrels of molasses could be filled from the contents of this tank?

16. How many cubic feet of earth will be removed in digging a well $3\frac{1}{2}$ feet in diameter and 10 feet deep?

VOLUME OF PYRAMID AND CONE

350. How do you find the volume of a prism? Of a cylinder?

Take a hollow prism and a hollow pyramid with the same base and altitude. Fill the pyramid with sand or water. Empty it into the prism. How many times do you repeat the operation to fill the prism?

Perform the same experiment with a cone and a cylinder of equal base and altitude.

If these measures are not at hand, you can easily make them from cardboard.

The volume of a prism or a cylinder = area of base × altitude.

The volume of a pyramid or a cone = approximately ⅓ area of base × altitude.

WRITTEN EXERCISES

351. Solve:

1. Find the volume of a cone 6 ft. high with a diameter at the base of 4 ft.

2. How many cubic feet are there in a monument in the shape of a triangular pyramid 20 ft. high, the base being 3 ft. on a side?

3. How many loads are there in a conical pile of sand 20 feet across at the base and 10 ft. deep? (1 load = 1 cu. yd.)

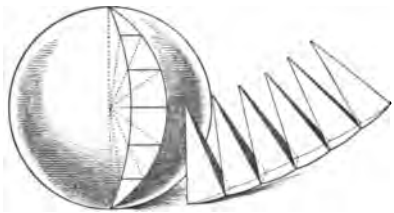
4. How many cubic feet of stone are required to build a solid steeple 40 ft. high with a square base 8 ft. on a side?

5. How many cubic feet are there in a conical pile of coal 150 ft. high and 125 ft. in diameter at the base?

6. How many cubic feet in a conical ant hill 2 ft. deep at the center, and 3 ft. across at the base?

VOLUME OF A SPHERE

352. As suggested in the picture, a sphere may be thought of as made up of a very large number of very small pyramids.



The sum of the bases of these pyramids equals the surface of the sphere.

The altitude of each of them is the radius of the sphere.

How do you find the volume of a pyramid?

How, then, do you find the volume of a sphere?

The volume of a sphere = ⅓ surface × radius.

The surface of a sphere $= 4\ \pi r^2$.

Then $\frac{1}{3}\ (4\ \pi r^2 \times r) =$ Volume.

The volume of a sphere equals ⅓ its radius × its surface.

WRITTEN EXERCISES

353. Solve:

1. Find the volume of a baseball 3 in. in diameter; a globe 2 ft. in diameter; 6 in. in diameter; 1 ft. in diameter.

2. What are the solid contents of a ball for bowling, 8 in. in diameter? 1 ft. in diameter?

3. How many times larger is a ball 3 in. in diameter than a ball 1 in. in diameter?

4. How many times larger is a 4-in. ball than a 1-in. ball? Than a 2-in. ball?

5. Find the volume of each of these balls.

REVIEW

354. Solve:

1. What is the approximate diameter of a circle whose circumference is 44″? (Use $\frac{7}{22}$ for the ratio.)

2. What is the area of a trapezoid whose parallel sides are 11″ and 17″, and the distance between them 10″?

3. What is the area of a right triangle whose base is 16″ and altitude 30″?

4. What is the area of an isosceles triangle whose base is 22″ and altitude 21″?

5. What is the approximate circumference of a circle whose diameter is 21″?

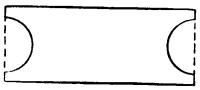
6. How many cubic feet of earth will be removed in digging a well $5\frac{1}{2}$ feet in diameter and 15 feet deep?

7. How thick is the trunk of a tree that is 7.854 feet in circumference?

8. Mr. Walsh has a square farm $\frac{1}{4}$ mile on a side. How many acres in the farm?

9. A circular lawn in a park is 120 feet in diameter. This has a border of geraniums $2\frac{1}{2}$ feet wide. How many yards of wire fencing will be required to inclose the geraniums on the outside?

10. (*a*) How many cubic feet of ensilage will a silo contain if it is 12 feet in diameter and 20 feet high? (*b*) How many bushels? (There are 2150.42 cubic inches in a bushel.)



11. Each of a class of 16 boys made a fish line reel.

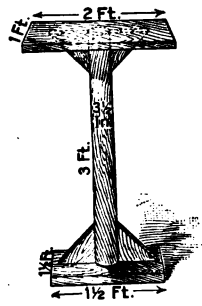
They used this drawing, which is on a scale of 4 in. to 1 in.

(*a*) How many square inches of wood were required?

(*b*) How many square inches were cut out of the ends of each reel? From all together?

What did the wood cost at $.04 a square foot?

A seventh grade manual training class made for the dictionary in the schoolroom a rectangular table, with a single round leg 3 ft. high, on a base.



The top of the table was 2×1 ft.

The base was $1\frac{1}{2} \times 1\frac{1}{2}$ ft.

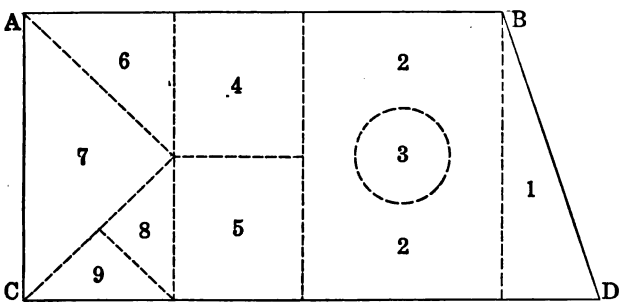
Four triangular brackets of wood were made, 6 in. on each side about the right angle, to support the top, and a like number to strengthen the base.

12. How many square feet of board were used in top, base, and brackets?

13. How many feet of board 1 foot wide were required to make the top, base, and brackets?

14. From a square piece of timber $3\frac{1}{2} \times 3\frac{1}{2}$ in., and of the required length, a round leg $3\frac{1}{2}$ in. in diameter and of the desired length was turned. How much wood was there in the timber? How much in the leg? How much was cut off?

How much did the wood for the table cost, boards being 5 cents a foot and the timber 5 cents a linear foot?



15. This diagram represents Mr. Gordon's farm, drawn on a scale of 1 inch to 80 rods. What figure is represented?

How does it differ from a rectangle?

How do you find the area of such a figure?

16. The line AB represents what fraction of a mile? The line CD? The line AC?

How many acres are there in the farm?

What kind of a figure is 1?

How do you find the area of such a figure?

17. What is its area?

18. The number of acres in this field is what per cent of the number of acres in the farm?

19. Mr. Gordon paid $14,437.50 for the farm. What was the average price per acre?

20. He paid 62½ per cent cash and gave a mortgage for the remainder. What was the amount of the mortgage?

21. He paid interest semiannually at 5 per cent per year. How much interest did he pay each time?

22. In the center of field 2 there is a lake ½ inch (by scale) in diameter. How many acres in the surface of the lake?

If the average depth of the water is 10 ft. in the lake, what is the approximate number of gallons in it, allowing 7.5 gallons to the cubic foot?

23. How many acres of land in field 2? What per cent of the farm is in the field?

24. What kind of a triangle does field 6 represent? What is its area?

25. What per cent is its area of the area of the farm?

26. Find the sum of the areas of all the fields.

27. What should this sum equal?

28. If the farm is assessed for 75 per cent of its value and taxed at the rate of $1.65 per hundred of assessed valuation, how much is the tax?

29. If the house on the farm is insured for $5,000 at the rate of $2\frac{1}{5}$ per cent per year, what is the premium?

30. Field 4 was a wood lot. Mr. Gordon cleared 43.5 per cent of the wood off the lot. How many acres of woods were left?

31. $12\frac{1}{2}$ per cent of the 1,200 trees still standing are oak, $66\frac{2}{3}$ per cent chestnuts, 15 per cent pine, and the remainder maples. How many trees were there of each kind?

32. Mr. Gordon sold a pile of oak wood 48 feet long, 4 feet wide, and 3 feet 6 inches high, for $6.75 per cord; a pile of chestnut 66 feet long, 4 feet wide, and 4 feet 6 inches high, for $4.87\frac{1}{2}$ per cord; and a pile of pine wood 30 feet long, 4 feet wide, 3 feet 9 inches high, for $5.25 per cord. How much did he receive for the wood?

33. In the southeast corner of the wood lot he fenced off a plot containing one acre, on which he built his house. This plot was 16 rods long. How wide was it?

34. He inclosed his house lot with a board fence 4.5 feet high, using 308 posts which were worth $18\frac{1}{2}$ cents each. The boards used were worth $35 per thousand feet. What was the value of the posts and boards used in the fence?

35. He built a silo 12 feet in diameter and 18 feet high. How many cubic feet of fodder would it hold?

36. He covered the bottom and sides with sheet iron. How many square feet were required?

SEEDS REQUIRED PER ACRE FOR PLANTING (APPROXIMATE)

	Bushels
Oats	$2\frac{1}{2}$
Rye	$1\frac{1}{2}$
Wheat	$2\frac{1}{2}$

37. What was the area of field No. 1?

38. Field 1 was sowed with oats. The cost of plowing was $3 an acre. The cost of the seed oats was 50 cts. per bushel. Fertilizer, costing $30 per ton, was used at the rate of 400 lb. to the acre. Labor for sowing and harrowing was $2 per acre; for harvesting, $1 per acre; for threshing, $1.25 per acre. The crop, 30 bushels per acre, was sold at 40 cts. per bushel. The straw averaged 1 ton per acre, and was sold at $10 per ton.

What was the profit on the oat field?

		Oat Field	*Dr.*	*Cr.*
Apr.	20	To plowing 15 A., at $3	45	
"	28	To 3 tons fertilizer, at $30	90	
"	29	To 37.5 bu. seed oats, at $.50	18 75	
"	30	To sowing and harrowing, at $2 per A.	30	
Aug.	10	To harvesting 15 A., at $1	15	
Oct.	11	To threshing 15 A., at $1.25	18 75	
Nov.	21	By 450 bu. of oats, at $.40		180
Dec.	12	By 15 tons of straw, at $10 per ton		150
		Balance,	112 50	
			330 00	330 00

39. Field 7 was sowed with rye.

The plowing cost $3 per acre.

The fertilizer cost $30 per ton, and 500 pounds were used per acre.

The seed rye cost 80 cts. per bushel (see table).

The labor for sowing, harrowing, and rolling cost $3 per acre.

The harvesting cost $1.50 per acre and the threshing $3 per acre.

The yield was 28 bushels per acre, which was sold for

80 cts. per bushel, and $1\frac{1}{4}$ tons of straw per acre which was sold for $16 per ton.

What was the profit on the rye field?

40. Make and balance the account for the rye field.

41. Stull Brothers, dealers in grain, hay, and feed, have a gasoline engine 35 horse power to run their mill. It takes one tenth of a gallon of gasoline for a horse power to run it an hour, at a cost of $.13 per gallon. What will the gasoline cost to run the engine 5 hr. 30 min.

42. If the mill grinds 400 pounds of corn in 5 minutes, how long will it take to grind a car load of 600 bushels? (Allow 56 lb. to the bushel.)

43. Lubricating oil, bought in quantities of 5 gallons, costs $.45 per gallon. If bought by the barrel of 50 gallons, it costs $.29, 10% off for cash and an allowance of $.75 for the barrel if returned. What is the cost per gallon, by the barrel, if all the above conditions are complied with?

44. What per cent is gained by purchasing by the barrel instead of in 5-gallon quantities?

45. The driving wheel of the engine makes 235 revolutions per minute, and that of the mill 1,750. The number of revolutions of the mill wheel is what per cent of the number of revolutions of the engine wheel?

46. Equal numbers of bushels of corn weighing 56 lb. per bushel and of oats weighing 32 lb. per bushel were ground together. The mixture weighed 5,280 lb. How many pounds of corn did the mixture contain? Of oats?

47. Mr. Baldwin has 275 bushels of potatoes, in bags of $2\frac{3}{4}$ bushels each. What are the potatoes worth at $2.25 per bag?

48. A grocer put 315 pounds of sugar in packages of $3\frac{1}{2}$ pounds each, which he sold at 23 cents per package. The sugar cost him $5\frac{1}{2}$ cents per pound. How much did he make?

49. The distance from New York to New London is 110 miles. How long will it take a steamer sailing at the rate of $18\frac{7}{8}$ miles an hour to go from New York to New London?

50. The product of the length and width of a corn crib is equal to $68\frac{1}{5}$ square feet; the cubic contents are $426\frac{1}{4}$ cubic feet. How high is the crib?

51. The length of a room is 30 feet, the width 25 feet, and the cubic contents 9,095 cubic feet. How high is the ceiling?

52. What is the value of 18 tons of coal, at the rate of $4\frac{1}{2}$ tons for $29.50?

53. If $37\frac{1}{2}\%$ of the number of cubic feet of air in a room is equal to 4,218.75 cubic feet, how many cubic feet of air are there in the room?

54. There are 40 pupils in this room. How many cubic feet of air does that give to each pupil?

55. A grocer sold 18 pounds 12 ounces of butter on Monday; $27\frac{3}{4}$ pounds Tuesday, 36.375 pounds on Wednesday. How many pounds did he sell? What is the value of the butter at $.26 a pound?

56. What is the sum of 16.5 feet, $25\frac{3}{4}$ feet, $42\frac{2}{8}$ feet, and 15 feet 6 inches?

57. Mr. Hawkins laid .375 of a mile of sidewalk. How much did it cost him at $13.20 a rod?

58. Mr. Wells bought a barrel of cranberries, containing 3 bushels, at $\$2\frac{1}{2}$ per bushel. He sold them at $12\frac{1}{2}$ cents per quart. How much did he make on the berries?

59. What is the diameter of a sphere whose surface is 31.416 square miles?

60. If a cannon ball weighs 36 pounds, what will one weigh whose diameter is three times as great?

61. A field in the form of a trapezoid contains 25 acres. One of its parallel sides is 90 rods, the other 105 rods. What is its width?

62. If a circus ring is 75 feet in diameter, how many times must a horse go around it to travel a mile?

63. What is the volume of a square pyramid, 6 feet on a side and having a slant height of 12 feet?

64. Compare the volume of the pyramid in **63**, with that of a cone having the same slant height, and base 24 feet in circumference.

65. How many 2-inch spheres does it take to equal in volume a 4-inch sphere? an 8-inch sphere?

SUMMARY OF CHAPTER VII

MENSURATION.

Measurement of Plane Figures.

Measurement of Solids.

Surfaces of Solids.

Volumes of Solids.

CHAPTER VIII

POWERS AND ROOTS

POWERS

354a. $2 \times 2 = 4$; $2 \times 2 \times 2 = 8$; $2 \times 2 \times 2 \times 2 = 16$.

$3 \times 3 = 9$; $3 \times 3 \times 3 = 27$.

In problem 1, how was 4 obtained? 8? 16?

In problem 2, how was 9 obtained? 27?

Note that in each case the product was obtained by multiplying together equal factors.

A product, or number produced by multiplying a number by itself, is called a **power** of that number.

The number itself is called the **first power.**

The product resulting from multiplying the number by itself once is called the **second power** or **square**; that resulting from multiplying the number by itself twice, the **third power** or **cube**; 4 is the second power of 2, and 8 is the third power of 2.

What is the second power of 3? Of 4? Of 6? Of 8?

What is the third power of 3? Of 4? Of 6? Of 8?

$2 \times 2 \times 2$ may be written 2^3.

3×3 may be written 3^2.

$5 \times 5 \times 5 \times 5$ may be written 5^4.

The small figures, 3, 2, and 4, are called exponents.

An **exponent** shows how many times the number is used as a factor. It is written at the right of the number and a little above it. $4^3 = 4 \times 4 \times 4$.

What does 3^5 equal? 6^3? 7^2? 8^3? 9^4?

355. POWERS OF NUMBERS TO 9

First power = 1, 2, 3, 4, 5, 6, 7, 8, 9.
Second power = 1, 4, 9, 16, 25, 36, 49, 64, 81.
Third power = 1, 8, 27, 64, 125, 216, 343, 512, 729.

Study this table until you can readily give the second and the third powers of the numbers in the first row.

What number multiplied by itself will produce 36?
What number multiplied by itself will produce 100?
What number multiplied by itself will produce 121?

ROOTS

356. What are the equal factors of 36? Of 49? Of 27? Of 8? Of 216? Of 64?

A number which is multiplied by itself to produce a given number, or which is one of the equal factors of the number, is called a **root** of that number.

If the root is one of two equal factors, it is called the **square root**; if it is one of three equal factors, it is called the **cube root.**

357. In the problem, $4 = 2 \times 2$, 2 is the square root of 4, because it is one of the two equal factors.

ORAL EXERCISES

358. In like manner find the square root of:

225	324	1,296	64	900	144
169	625	121	196	256	400

359. In the problem, $8 = 2 \times 2 \times 2$, 2 is one of the three equal factors and is the cube root of 8.

Find the cube root of the following numbers by finding one of their three equal factors:

343, 216, 1,000, 1,728, 2,744, 512.

360. Instead of writing the words square root, cube root, etc., mathematicians use a sign to indicate roots. They use the sign $\sqrt{}$, $\sqrt[3]{}$, $\sqrt[4]{}$. It is called the **radical**, or root sign. When it is used without the figure at the opening, the square root is indicated; when the figure 3 is used, the cube root is indicated. In arithmetic, roots above the cube root are not considered.

$\sqrt{9}$ indicates the square root of 9.

$\sqrt[3]{64}$ indicates the cube root of 64.

361. The method of finding the square root and the cube root of numbers by means of equal factors is of use when the numbers can be separated into equal, integral factors.

The following method enables one, in many cases, to determine at a glance the square root of a number. It grows out of the method of multiplying in which every complete product is put down, instead of adding and combining as one multiplies.

ILLUSTRATIONS:

$$\begin{array}{r} 12 \\ \underline{12} \\ 4 = \text{units squared} = u^2. \\ 20 = \text{tens} \times \text{units} = t \times u. \\ 20 = \text{tens} \times \text{units} = t \times u. \\ \underline{100} = \text{tens squared} = t^2. \\ 144 = \text{units squared} + 2\ (\text{tens} \times \text{units}) + \\ \qquad \text{tens squared} = u^2 + 2(t \times u) + t^2. \end{array}$$

$$\begin{array}{r} 16 \\ \underline{16} \\ 36 = u^2. \\ 60 = t \times u. \\ 60 = t \times u. \\ \underline{100} = t^2. \\ 256 = u^2 + 2(t \times u) + t^2. \end{array} \qquad \begin{array}{r} 24 \\ \underline{24} \\ 16 = u^2. \\ 80 = t^2. \\ 80 = t \times u. \\ \underline{400} = t \times u. \\ 576 = u^2 + 2(t \times u) + t^2. \end{array}$$

FINDING ROOTS

THE MULTIPLICATION METHOD

362. These multiplications teach us how to proceed without factoring if we wish to find the square root of 144, or the number which multiplied by itself makes 144.

It will be seen that 144 is made up of two squares, $10^2 + 2^2$, and twice the product of the numbers making the squares, $2 \times 10 \times 2$. It can be separated into these parts as follows:

ILLUSTRATIONS: $144 = 100 + 4 + 40$.
The square root of $100 = 10$
The square root of $4 = 2$
Their sum $= 12$, the square root.

TEST: $10^2 = 100$
$2^2 = 4$
$2 \times 10 \times 2 = 40$
Total $= 144$

$169 = 100 + 9 + 60$.
$\sqrt{100} = 10$
$\sqrt{9} = 3$
Their sum $= 13$, the square root.

TEST: $10^2 = 100$
$3^2 = 9$
$2 \times 10 \times 3 = 60$
Total $= 169$

$625 = 400 + 25 + 200$.
$\sqrt{400} = 20$
$\sqrt{25} = 5$
Their sum $= 25$, the square root.

TEST: $20^2 = 400$
$5^2 = 25$
$2 \times 20 \times 5 = 200$
Total $= 625$

$6889 = 6400 + 9 + 400$.
$\sqrt{6400} = 80$
$\sqrt{9} = 3$
Their sum $= 83$, the square root.

TEST: $80^2 = 6400$
$3^2 = 9$
$2 \times 80 \times 3 = 480$
Total $= 6889$

WRITTEN EXERCISES

363. Find the square root of the following numbers by the method just given:

1. 361. **2.** 729. **3.** 7,225. **4.** 5,929. **5.** 1,296. **6.** 1,681.

364.

$$1 \times 1 = 1$$
$$9 \times 9 = 81$$
$$10 \times 10 = 100$$
$$99 \times 99 = 9{,}601$$
$$100 \times 100 = 10{,}000$$
$$999 \times 999 = 998{,}001$$

9, the largest number expressed by one figure, when squared, gives 81, a number of two figures.

10, the smallest number expressed by two figures, when squared, gives a number of three figures; and 99, the largest number expressed by two figures, when squared, gives 9,601, a number of four figures.

100, the smallest number expressed by three figures, when squared, gives 10,000, a number of five figures; and 999, the largest number expressed by three figures, when squared, gives 998,001, a number of six figures.

Therefore, if the number of which the square root is to be found contains one or two figures, there will be one figure in the root; if it contains three or four figures, there will be two figures in the root; if it contains five or six figures, there will be three figures in the root.

ORAL EXERCISES

365. How many figures will there be in the square root of each of the following numbers?

1. 9.	**3.** 225.	**5.** 2,597,586.	**7.** 2,965.
2. 64.	**4.** 46,765.	**6.** 34,758,267.	**8.** 868,675.

366. The first step in extracting the square root of a whole number is to divide it into periods of two figures each, beginning at the right.

In the numbers that you have produced by squaring other numbers, you will observe that the tens squared can always

be found in the left-hand period. If you can determine the square, you can find what the tens' figure is by extracting the square root of this square. For example, in the number 1,024, the greatest square is 900. The square root of 900 is 30, which represents the tens.

ORAL EXERCISES

367. Find the greatest squares and the tens' figure in each:

1. 121.	3. 5,929.	5. 9,601.	7. 348,767.
2. 2,116.	4. 14,641.	6. 21,345.	8. 3,446.

368. In the following, after finding the greatest square and its root, divide the number by twice the root and find in the quotient the other root from the formula $u^2 + 2 \times u \times t + t^2$.

169, 576, 1089, 256, 625, 196, 2025, 361, 2500, 1296, 4096.

Extract the square root of 1,024.

SOLUTION:

$30 + 2 = 32$, the square root.

$t^2 + 2(t \times u) + u^2 = 1{,}024$

$t^2 = 900$, the greatest square in the left-hand period.

$t = 30$	
$2t = 60$	
$60 \times u + u^2 =$	124
$u = 2$	
$60 \times 2 + 2^2 =$	124

Extract the square root of 2,116.

SOLUTION:

$40 + 6 = 46$, the square root.

$t^2 + 2(t \times u) + u^2 = 2{,}116$

$t^2 = 1{,}600$, the greatest square in the left-hand period.

$t = 40$	
$2t = 80$	
$80 \times u + u^2 =$	516
$u = 6$	
$80 \times 6 + 36 =$	516

Extract the square root of 6,889.

SOLUTION: $80+3=83$, the square root.

$t^2+2(t\times u)+u^2=6,889$

$t^2=6,400$, the greatest square in the left-hand period.

$t=80$	
$2\,t=160$	
$160\times u+u^2=$	489
$u=3$	
$160\times 3+3^2$	489

Extract the square root of 55,225.

SOLUTION: $200+30+5=235$, the square root.

$t^2+2(t\times u)+u^2=55,225$

$t^2=40,000$, the greatest square in the left-hand period.

$t=200$	the portion of the root already found.
$2\,t=400$	
$400\times u+u^2=$	15,225
$u=30$	
$400\times 30+30^2=$	12,900
$t=230$	the portion of the root already found.
$2\,t=460$	
$460\times u+u^2=$	2,325
$u=5$	
$460\times 5+5^2=$	2,325

WRITTEN EXERCISES

369. Extract the square root of the following numbers:

1. 6,724.
2. 60,516.
3. 374,544.
4. 23,104.
5. 4,624.
6. 591,841.

If the number of which the square root is to be found is a decimal, begin at the decimal point and divide into periods, both right and left. If necessary, add a cipher to the right

hand period of the decimal so that each period may have two figures. Then proceed as before.

When it is required to extract the square root of a common fraction, extract the square root of the numerator and denominator separately if they are perfect squares. If they are not squares, change the fraction to a decimal before finding the square root.

When it is required to extract the cube root of a common fraction, extract the cube root of the numerator and denominator separately if they are perfect cubes. If they are not perfect cubes, change the fraction to a decimal before finding the cube root.

WRITTEN EXERCISES

370. Solve:

1. $\sqrt{2.56}$.	**5.** $\sqrt{36.3}$.	**9.** $\sqrt{.001764}$.	**13.** $\sqrt{3.285}$.
2. $\sqrt{56.25}$.	**6.** $\sqrt{\frac{4}{49}}$.	**10.** $\sqrt{401.2009}$.	**14.** $\sqrt{\frac{1}{2}}$.
3. $\sqrt[3]{64}$.	**7.** $\sqrt[3]{125}$.	**11.** $\sqrt[3]{\frac{1}{27}}$.	**15.** $\sqrt[3]{\frac{8}{27}}$.
4. $\sqrt{\frac{1}{8}}$.	**8.** $\sqrt[3]{\frac{1}{64}}$.	**12.** $\sqrt[3]{216}$.	**16.** $\sqrt[3]{\frac{64}{125}}$.

17. How many rods of fence will it require to fence one side of a square field containing 10 acres?

18. Find the square root of 1.21.

371. THE GEOMETRIC METHOD.

Draw a line 5 inches long. See diagram, p. 228.

Draw a square upon it.

How many square inches in the square?

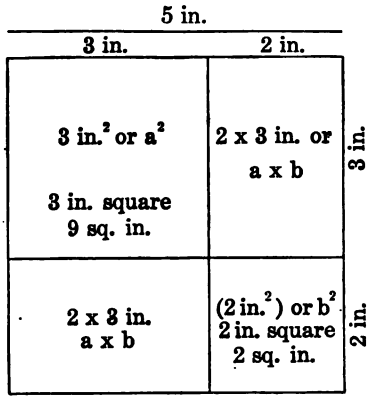
$5 = 3 + 2$.

What is the square of 3? Of 2?

How much less than 5^2 is $3^2 + 2^2$?

Draw and cut out a 3-inch square; a 2-inch square.

Place them on the 5-inch square, as shown in the diagram.



Call 3 inches a, and 2 inches b.

How large is each of the sections of 5 in.^{2} not covered by 2 in.^{2} and 3 in.^{2}?

We see that the squares of two numbers taken together are not equal to the square of their sum.

As each section left over is 2×3 square inches, their sum is $2 \times (2 \times 3)$ square inches.

That is, 5 in.^{2} = 2 in.^{2} + 3 in.^{2} + $2 \times 2 \times 3$ in.

Or, the square of 2 and 3 and twice the product of 2 and 3 = the square of 5.

Since 3 in. = a and 2 in. = b, then $(a + b)^2 = a^2 + b^2 + 2\,ab$.

Prove it by finding the square of $a + b$ by multiplication.

The square of the sum of two numbers is equal to the sum of the squares of the numbers plus twice their product.

$15 = 10 + 5$. Then $15^2 = 10^2 + 5^2 + 2 \times 5 \times 10 = 225$.

Find the square of 16.

$16^2 = 10^2 + 6^2 + 2 \times 10 \times 6$.

Find at sight the squares of 17, 18, 19, 21, 22, 23, 24, 25.

372. The square root of a number too large to be factored at sight is found by reversing the process of squaring given above; that is, by separating the number into two smaller squares, as a and b, and finding the roots of these separately, making allowance for the $2\,ab$, or twice the product of the roots.

Find the square root of 196.

Since 196 is larger than the square of 10, it is evident that

the square root contains two figures, a unit's figure and a ten's figure. Call the ten's figure a and the unit's figure b.

Then 196 = the square of $(a + b) = a^2 + 2\,ab + b^2$.

a^2 = the ten's figure squared.

b^2 = the unit's figure squared.

$2\,ab$ = twice the product of $a \times b$.

$196 = 100 + 96 = 100 + 16 + 80$.

$100 = a^2$; $16 = b^2$; $80 = 2\,ab$.

100 is the square of 10; and 16 is the square of 4.

$2 \times 4 \times 10 = 80$.

The square root of 196 is $10 + 4 = 14$.

As we cannot always tell at sight the parts into which a number may be divided for the purpose of extracting the square root, we simplify the process by dividing it into periods which suggest the proper division.

Thus, in finding the square root of 225, since 225 is larger than the square of 9, we can see that there must be in the root a ten's and a unit's figure. So we mark it thus, 2/25, or thus, 2 $\acute{2}5$. This means 200 + 25. The 200 must contain a^2, the square of the ten's figure of the root. The square of 20 (two tens) is not contained in 200. So the largest ten whose square is contained in 200 is 10 (1 ten).

Taking 10^2 from 225 leaves 125.

$225 - 100 = 125$; $a^2 + b^2 + 2\,ab - a^2 = b^2 + 2\,ab$.

125 must contain the square of the unit's figure, plus $2 \times$ the product of the unit by the ten's figure.

To find the unit's figure, we divide 125 $(2 \times 10 \times b)$ by (2×10).

$$\frac{(b^2 + 2ab)}{2a} = \frac{125}{20} = 5 \text{ with a remainder of } 25;\ 25 = 5^2.$$

Hence $125 = 20 \times 5 + 5^2$ $(2a \times b + 5^2)$.

That is, $225 = 10^2 + 2 \times 10 \times 5 + 5^2 = (10 + 5)^2$, and $\sqrt{225} = 10 + 5 = 15$.

Observe that $\frac{125}{20} = 6\frac{5}{20}$; that is, the quotient is 6 rather than 5. The reason we use 5 is this: 125 contains not only $2ab$, but b^2 besides $(2ab + b^2)$. Hence allowance must be made for the b^2, the product of the unit's figure by itself.

We commonly do not put down the cipher in the unit's place of either the quotient, or the divisor, but leave the place vacant and write in it the unit's figure when found. Thus:

$$\begin{array}{r} 225(1 \\ 100 \\ \hline 2)125 \end{array}$$

The Process:

$$\begin{array}{r} \,,\;, \\ 2\;25(1 \end{array}$$

We divide 225 into 2 periods, 2 (hundred) and 25.

We say: The greatest square in 2 is 1. Put 1 in the root. Subtract its square, 1, from 2.

$$\begin{array}{r} 225(15 \\ 1 \\ \hline 25)125 \\ 125 \\ \hline \end{array}$$

Double 1 for a trial divisor and divide.

It appears to go 5 times. Annex 5 to 2 and also place it in the root and multiply. 15 is the square root of 225.

1. To find the square root, beginning at the right, divide the number into periods by marking every alternate figure. (If the number is or contains a decimal, begin at the point.)

2. Find the greatest square in the first, or left-hand period. Place its square root in the quotient, subtract the square, and bring down the next period.

3. Double the root found for a trial divisor. Find a trial quotient and place it in the unit's place of the quotient, and also of the trial divisor.

4. Multiply the trial divisor by the trial quotient and subtract the product. (If the subtrahend is larger than the minuend, try again.) Bring down the remainder, if any, and proceed as before.

WRITTEN EXERCISES

373. Solve:

1. What is the length of 1 side of a square rug containing 64 square yards?

2. Our athletic field is square and contains 16 acres. How many rods long is one side?

3. In a square field there are 625 square rods. What is the length of one side of the field in rods?

4. Mr. Williams has a square field containing 400 square rods. How long is each side of the field? What is the perimeter of the field?

5. Mr. Thurston has a cubical box containing 216 cubic feet. How long, wide, and high is the box? How many square inches in the surface of the box? How many inches in the edges of the box?

6. A farmer has a bin whose length, breadth, and depth are the same. There are 729 cubic feet in the bin. What are its dimensions?

7. How many square feet of boards did it take to build the bin with a cover?

8. There are 1,000 cubic feet in a small room whose three dimensions are the same. How high is the ceiling of the room? How many square feet in the floor?

9. The number of square feet in the floor of a room is 225. The length and the breadth are the same. What are its dimensions? What is the perimeter of the room? How many yards in the perimeter?

10. Mr. Hudson dug a cellar; the length, breadth, and depth were the same. There were 8,000 cubic feet in the cellar. What did it cost to cement the cellar at 5 cents per square foot?

RIGHT TRIANGLES

374. Draw a square on the scale of $\frac{1}{3}$ in. to 1 in. to represent a 15-in. square.

a^2	a x b
a x b	b^2

Divide it as the square is divided and write in each section the number of square inches it contains and the formula representing it.

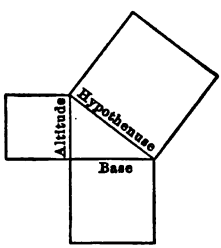
In Section 3 of the rule, why is the root already found doubled to make the trial divisor?

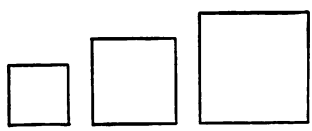
What part of the square of 15 is shown by a^2? b^2?

375. To find the Hypotenuse of a Right Triangle.

The side of a right triangle opposite the right angle is called the **hypotenuse.**

Draw a right triangle on a base 4 inches long, with an altitude of 3 inches.





Measure the hypotenuse.

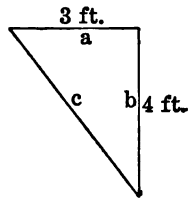
Draw squares on lines equal to the base, altitude, and hypotenuse of the triangle.

How many square inches in each square?

Compare the sum of the squares of the base and altitude with the square of the hypotenuse.

Calling the base b, altitude a, and hypotenuse h, we have equation,

$$a^2 + b^2 = h^2; \text{ or}$$
$$a^2 = h^2 - b^2;$$
$$b^2 = h^2 - a^2.$$



In the above triangular bracket what is the length of c?

$$a^2 + b^2 = c^2$$
$$9 + 16 = c^2$$
$$c^2 = 25$$
$$c^2 = \sqrt{25} = 5$$

WRITTEN EXERCISES

376. Solve:

1. Find the missing side of each of the following triangles:

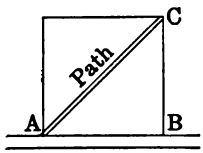
	ALTITUDE	BASE	HYPOTENUSE
(1)	3		5
(2)		6	8
(3)	5	12	
(4)	13		14

2. A pole is 12 ft. high. It is supported by a guy rope fastened to a stake 9 ft. from its foot. How long is the guy rope?

3. A ladder is 17 ft. long. To reach a window 15 ft. from the ground, how far from the base of the wall must the foot of the ladder be placed?

4. A man fell into the water from a pier 15 ft. above the water. A rope was thrown to him. It required 20 ft. of rope to reach him. How far was he away from the pier?

5. Mr. Simmons' ice house has an inclined shoot for taking in ice. It extends from a door 20 ft. above the ground to the edge of the pond, 60 ft. from the building. How long is the shoot?



6. The lot at the corner of 2d St. and Fourth Ave. is 120 × 50 ft. A path ran diagonally across it. The owner built upon it, closing the path. How much farther must people now go to get from A to C?

CUBICAL CONTENTS

377. How many dimensions must you know in order to calculate the contents of a parallelopiped?

What are the factors? What is the multiple?

The contents of a bin are 9,800 cubic feet; the length 28 feet, and the width 25 feet. What is the height?

$$28 \times 25 = 700;\ 9{,}800 \div 700 = 14.$$

When the cubic contents and two of the dimensions of a parallelopiped are given, the third dimension is found by dividing the cubic contents by the product of the given dimensions.

WRITTEN EXERCISES

378. Solve:

1. There are 8,424 cubic feet in a room. The length of the room is 27 feet; the width is 24 feet. What is its height?

2. Mr. Doremus dug a cellar, from which he removed 9,180 cubic feet of dirt. The length of the celler was 36 feet; the depth was 8.5 feet. What was the width of the cellar?

3. What did it cost to dig the cellar at 12½ cents per cubic yard? (A cubic yard is 3 feet long, 3 feet wide, and 3 feet high.)

4. There were taken from a marl pit 1,560 cubic feet of marl. The width of the pit was 8 feet; the depth was 12 feet. How long was it?

5. What was the value of the marl at $1.25 per load, a load containing 40 cubic feet?

Find the missing dimensions of the following:

6. Contents 240 cubic feet, length 8 feet, width 5 feet.

7. Contents 880 cubic feet, length 11 feet, depth 8 feet.

8. Contents 1,880 cubic feet, width 10 feet, depth 9 feet.

9. Contents 3,200 cubic feet, length 25 feet, width 16 feet.

10. Contents 8,400 cubic feet, length $33\frac{1}{3}$ feet, width 21 feet.

11. Contents 4,500 cubic feet, length 18 feet, width $16\frac{2}{3}$ feet.

SUMMARY OF CHAPTER VIII

POWERS AND ROOTS — APPLICATIONS.
- Powers.
- Roots by method of factoring.
- Methods of finding roots.
  - Multiplication Method.
  - Geometric Method.

APPLICATIONS OF SQUARE AND CUBE ROOT.
- In finding the sides of a right triangle.
- In finding the dimensions of a cube.

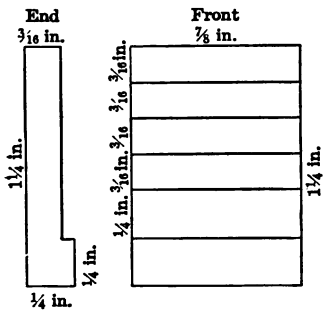
GENERAL REVIEW

379. Solve:

1. From a farm containing 135 acres 60 square rods, there were sold 72 acres 100 square rods. How many acres remained?

2. A man has purchased three varieties of strawberry plants; of the first he has 210 plants, of the second 168 plants, and of the third 231 plants. He wishes to set them in rows, each variety by itself, so that there will be the same number of plants in each row. What is the greatest number that he can put in a row? How many rows of each variety will there be?

3. In their manual training class, the eighth grade made a set of bookshelves for the schoolroom. The teacher gave them drawings, on a scale of $\frac{1}{4}$ in. to 1 ft., like the copies given here. Measure these copies and make estimates as follows:



Number of board feet in the sides.

Number of board feet in the back.

Number of board feet in the shelves.

If boards 13 inches wide are bought, how much is wasted?

4. At $\$.05\frac{1}{2}$ a square foot, what did the wood cost?

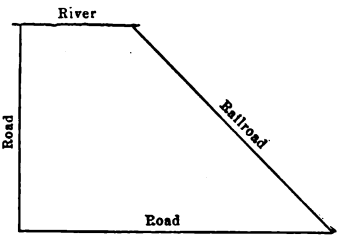
5. What width of board would be better to use? How much less would it cost?

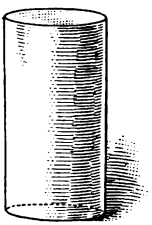
6. How many books $1\frac{3}{4}$ inches thick could be placed on the shelves?

7. The Equator is 24,899 miles long. How far is it to the center of the earth from a point on the Equator? How far through the earth?

8. New York is almost exactly opposite the island of St. Paul in the Pacific Ocean south of the Equator. How far is it in a direct line on the surface of the earth from New York to St. Paul Island? How far through the earth?

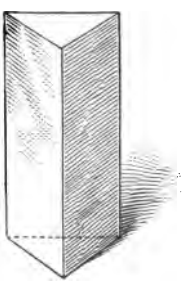
9. This is a diagram of Mr. Preston's farm. The longer road extends along by the side of the farm for a distance of 960 rods, the shorter one for a distance of 720 rods. The side of the farm that is bordered by the river measures 360 rods. Find the length of the side bordered by the railroad. How many acres are there in the farm?





10. This cylinder is drawn on a scale of 5 feet to an inch. Find the area of the curved surface. What is the area of the bases?

11. This figure represents a stone that is 4 feet long and whose equilateral base is $1\frac{1}{2}$ feet high. Find the number of cubic feet in the stone.



12. How many square feet of tin will be required to line the flat roof of a circular summer house having a circumference of 68.54 feet?

13. How many acres are there in a circular park having a diameter of 120 rods?

14. A California redwood tree measures 17.2788 feet around the trunk. What is the diameter of the tree? What is the area of a cross section?

15. Philadelphia is 75° west of London. What is the difference in time between the two cities?

16. New York is 74° 3′ west longitude and Chicago is 87° 38′ west. What is the difference in time?

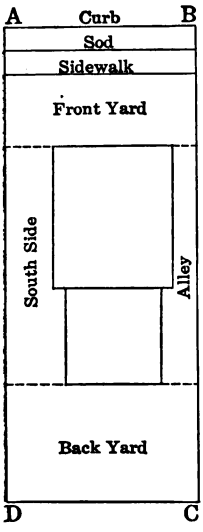
17. St. Petersburg is 30° east longitude. Washington is 77° west. When it is 9 A.M. in Washington, what time is it in St. Petersburg?

18. When it is 8 P.M. at St. Petersburg, what time is it in Washington?

19. Mr. Wilson had a winter cover for his fountain made in the shape of a square pyramid 6 feet on a side with a slant height of 10 feet. How many feet of boards did it take?

20. The upright post of a derrick 20 feet high is supported by guy cables attached to its top and fastened at the ground to stakes 20 feet from the base of the post. How long are the cables?

21. The rectangle $ABCD$ represents a building lot. It is drawn on a scale of 80 feet to 1 inch. What are its dimensions?



22. The figure within the rectangle represents the ground floor of the house, drawn on the same scale. How wide is the front of the house? How wide is the rear of the house? How long is the house? How many square feet does it cover?

23. The sidewalk is made of paving stone 5 feet wide, that cost \$.14 a square foot, laid. What did the sidewalk cost?

24. The remaining part of the front lot is sodded. The sod cost \$.25 per square yard, laid. What did it cost to sod the front yard?

25. The alley is paved with brick and stone at an average cost of \$.13½ per square foot. What did it cost to pave the alley?

26. The south side and back yard are sodded. The sod cost $.25 per yard laid. What did it cost to sod them?

27. The fence on the south and west sides cost, on the average, $.18¾ per foot. What was the cost of the fence if it was built to the front of the house?

28. The cellar wall extends entirely around the house. It is 7½ feet high and 18 inches thick. How many cubic feet in the wall? Allowing 24 bricks to a cubic foot, how many thousand bricks in the wall?

29. The carpenter's contract for building the house was $3,694. The mason's contract was $1,340, the plumber's $325. The electric wiring cost $110. What was the amount of the contracts for the house?

30. The carpenter's work was what per cent of the amount of the contracts? The mason's? The plumber's? The painter's?

31. The mason's contract was what per cent of the carpenter's? The plumber's contract was what per cent of the mason's?

32. The drawing-room on the second floor is 25 feet long and 13 feet 6 inches wide. What did the floor boards cost at $45 per thousand feet? Laying the breadths across the room, what would it cost to carpet the room with Brussels carpeting ¾ yard wide at $2.25 per yard, allowing 2½ % for matching?

33. The bedrooms on the southeast corner in the rear of the house are 15 feet long, 13 feet 6 inches wide, and the walls are 9 feet high. Allowing 12½ % for doors and windows, what would it cost to plaster each room at $.30 per square yard?

34. On the walls of the house 300 rolls of paper were used, at an average marked price of $.40 per roll. The paper was bought for 5 %, 10 %, and 15 % off for cash. What was the cost of the paper?

35. 1150 feet of molding were furnished. The average price was \$.05 a foot, with 10 % off for cash. What was the cash price for the molding? If it cost \$.01½ a foot to put on the entire molding, what was the entire cost?

36. An allowance of 25 % was made on 2 mantels for which \$75 each was asked. What was the cash price of the mantels?

37. The first payment to the carpenter was 25 % of his contract price, due when the building was inclosed. The man building the house had his note discounted at 5 % for 30 days to get the money to pay for it. What were the proceeds of the note?

38. The second payment, 33⅓ % of what was still due on the contract, was paid when the partitions were up and the floors were laid. How much was it?

39. The third payment, 50 % of what still remained to be paid on the contract, was paid when the house was trimmed. How much was it?

40. When the carpenter's work was completed, the owner gave a 90-day note at 5 % for the amount still due to the carpenter. What was the amount of the note when due?

41. According to the items given, what was the entire cost of the house?

42. The cost of the lot was \$1,200. Allowing \$25 a year for water, \$15 a year for insurance, \$75 for repairs, and 5 % for the use of the money invested, what will be the net profit for a year if the house is rented at \$80 per month?

43. The ball on an engine-house tower is 2 feet 6 inches in diameter. How many square inches of gilding will be required to gild it?

44. What will it cost to paint a smokestack 15 feet high and 9 feet in circumference at \$.04 per square foot?

45. A square field contains 9 acres. What is the length of its diagonal?

46. Mr. Baker had a farm having two parallel sides, one $\frac{1}{2}$ of a mile and the other $\frac{5}{16}$ of a mile long. The distance between them was $\frac{1}{4}$ of a mile. Draw to scale, 1 inch to 80 rods.

47. How many acres were there in the farm?

48. Letter the short parallel side AB and the longer side CD, A and C being the vertices of the right angles. Connect BC, thus dividing the farm into two parts. In ABC wheat was planted; in BCD corn was planted. Find the number of acres in the wheat field.

49. Find the number of acres in the corn field.

50. In a park there is a circular fountain 35 feet in diameter. Encircling this is a gravel walk 7 feet wide. What is the distance around the outer edge of the walk? What is the area of the fountain?

51. A race course is half a mile in circumference. How many feet is it across the course?

52. Mr. Small bought $33\frac{5}{6}$ acres of land at $\$42\frac{7}{8}$ an acre. How much did the land cost him?

53. He sold the land for $\$57\frac{5}{9}$ an acre. How much did he gain?

54. If it takes $4\frac{1}{4}$ yards of cloth to make a coat, $1\frac{3}{4}$ yards to make a vest, and $3\frac{3}{4}$ yards to make a pair of trousers, how many yards are required for the suit?

55. Mrs. Doane bought $3\frac{3}{4}$ pounds of rice at \$.08 a pound, $2\frac{1}{2}$ pounds of tea at \$.58 a pound, 20 pounds of sugar at $\$.04\frac{3}{4}$ a pound, and $1\frac{3}{8}$ pounds of cheese at \$.22 a pound. What was the entire amount of her bill?

56. Mr. Green paid \$1,680 for 16 acres of land. How much would he pay for 12 acres at the same rate?

57. From a barrel of gasoline, a dealer sold 5 gal. 3 qt. to one man and 10 gal. 2 qt. to another. How many gallons remained?

58. Mr. Doremus sold 52 bu. of wheat, which was 13 % of his whole crop. How many bushels had he left? What was its value at $.87\frac{1}{2}$ a bushel?

59. Mr. Pierson's house is insured for $4,500 and his furniture for $800. The insurance on the house is 60 % of its value, and that on the furniture is $66\frac{2}{3}$ % of its value. What is the value of the house and furniture?

60. In 35 % of an acre there are 15,246 sq. ft. How many square feet are there in an acre?

61. From a cask 15 gallons of molasses were drawn, which was 24 % of its contents. What was the value of the whole cask at $.75 per gallon?

62. What is the gain or loss per cent on an article bought for $\frac{1}{4}$ of a dollar and sold for $.37\frac{1}{2}$?

63. A man increased his money $37\frac{1}{2}$ %. He then had $121. How much had he at first?

64. Mr. Ames paid $8,260 for a house, which was $3\frac{1}{3}$ % of his money. How much money had he?

65. Mr. Morgan lost $45,500, which was $2\frac{1}{2}$ % of his money. How much had he at first?

66. A bookseller sold 2 books for $.96 each. On one he made 20 % and on the other he lost 20 %. Did he gain or lose on both, and what per cent?

67. Three men engaged in business. Mr. Tuttle put in $7,250 which was $6\frac{1}{4}$ % of his money; Mr. Rice put in $9,500, which was $14\frac{2}{7}$ % of his money; and Mr. Ludlow put in $20,500, which was $83\frac{1}{3}$ % of his money. How much money had each? How much money had they together?

68. A speculator bought a tract of land containing a square mile at $24 an acre. He sold it to Mr. Davis at a profit of $16\frac{2}{3}\%$. Mr. Davis divided it into farms of 160 acres each and sold each farm for $5,000. How much did Mr. Davis make? How much did the speculator make?

69. A wholesale dealer in jewelry sold a watch that cost him $80 to a retail dealer at a profit of 40%. The retail dealer sold it to a customer for 30% more than it had cost him. Which man made the more. How much more?

70. Mr. Andrews bought 4,500 lb. of Timothy hay at $\$.0\frac{3}{4}$ per pound, and sold it for $20 per ton. What was his gain per cent?

71. Helen's age is 6 yr. 4 mo. 20 da. which is $12\frac{1}{2}\%$ of her father's age. How old is her father?

72. Mr. Durand and Mr. Lawson each bought a tract of land, paying the same price. Mr. Durand sold his tract for $7,200 and made 25%. Mr. Lawson sold his tract for $6,336. How much did he make! What per cent did he make?

73. Find the difference between $12\frac{1}{2}\%$ of 4,864 feet and $\frac{1}{8}\%$ of 48,640 feet.

74. Find the difference between $37\frac{1}{2}\%$ of 9,728 minutes and $\frac{3}{8}\%$ of 97,280 minutes.

75. Mr. Sartolia sold 25 bu. 3 pk. 6 qt. of peanuts. This was $6\frac{1}{4}\%$ of all that he had. How many bushels did he have at first?

76. If John had 25% more money than he has, he would have $400. How much has he?

77. Jan. 1, 1908, Mr. Riley borrowed $360 of Mr. Hawkins, agreeing to pay him 5% for the use of the money. How much did Mr. Riley owe Mr. Hawkins Aug. 16, 1909?

78. What is the interest on $480 at 6% from February 12, 1907, to April 18, 1908?

SUPPLEMENT

INTERNATIONAL METRIC SYSTEM

SYNOPSIS OF THE SYSTEM

1. The fundamental unit of the Metric System is the **meter** — the unit of length. From this the units of capacity (**liter**) and of weight (**gram**) were derived. All other units are the decimal subdivisions or multiples of these.

These three units are simply related; *e.g.* for all practical purposes one **cubic decimeter** equals one **liter** and one **liter** of water weighs one **kilogram.**

The metric tables are formed by combining the words "**meter,**" "**gram**" and "**liter**" with the six numerical prefixes, as in the following tables.

The tables and descriptions in the following are taken from the pamphlet on the Metric System issued by the Department of Commerce and Labor of the United States government.

Prefixes	Meaning		Units
milli- = one thousandth	$\frac{1}{1000}$	.001	
centi- = one hundredth	$\frac{1}{100}$	.01	"meter" for length
deci- = one tenth	$\frac{1}{10}$	.1	
unit = one		1	"gram" for weight or mass.
deka- = ten	$\frac{10}{1}$	10	
hecto- = one hundred	$\frac{100}{1}$	100	"liter" for capacity
kilo- = one thousand	$\frac{1000}{1}$	1000	

2. UNITS OF LENGTH

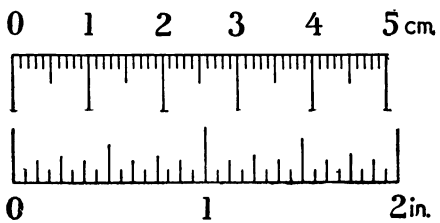
milli-meter	.001	meter,	mm.
centi-meter	.01	"	cm.
deci-meter	.1	"	dm.
meter	1	"	m.
deka-meter	10	"	Dm.
hecto-meter	100	"	Hm.
kilo-meter	1000	"	Km.

Where miles are used in England and the United States for measuring distances, the kilometer (1,000 meters) is used in metric countries. The kilometer is about 200 rods.

There are about 1,600 meters in a statute mile, 20 meters in a chain, and 5 meters in a rod. One meter = 39.37 inches exactly.

3. The **meter** is used for dry goods, merchandise, engineering construction, building, and other purposes where the yard and foot are used. The meter is about a tenth longer than the yard.

The **centimeter** and **millimeter** are used instead of the inch and its fractions in machine construction and similar work. The centimeter, as its name shows, is the hundredth part of a meter. It is used in cabinet work, in expressing sizes of paper and books, and in many cases where the inch is used. The centimeter is about two-fifths of an inch and the millimeter about one twenty-fifth of an inch. The millimeter is divided for finer work into tenths, hundredths, and thousandths.



4. If a number of distances in millimeters, meters, and kilometers are to be added, reduction is unnecessary. They are added as dollars, dimes, and cents are now added. For example, "1,050.25 meters" is not read "1 kilometer, 5 dekameters, 2 decimeters, and 5 centimeters," but "one thousand and fifty meters, twenty-five centimeters," just as "$1,050.25" is read "one thousand and fifty dollars, twenty-five cents."

Complete the table:

—— mm. = 1 ——
—— cm. = 1 ——
—— dm. = 1 ——
—— m. = 1 ——
—— Dm. = 1 ——
—— Hm. = 1 ——

ORAL EXERCISES

5. Solve:

1. What is the ratio of 1 Km. to 1 Hm.? To 1 Dm.? To 1 M.? How many Dm. in 6 Km.? In 8 Km.? In 4.5 Km.? In 9.25 Km.? In 12.75 Km.?

2. How many M. in 1 Km.? In 5 Km.? In 7 Km.? In $7\frac{1}{2}$ Km.? In $4\frac{1}{4}$ Km.? In $7\frac{3}{4}$ Km.?

3. How many M. in 2 Dm. 5 M.? In 9 Dm. 6 M.? In 8 Dm. 4 M.? In 1 Hm. 1 Dm. 2 M.? In 5 Hm. 3 Dm. 6 M.? In 6 Km. 5 Hm. 4 Dm. 3 M.? In 12 Km. 8 Hm. 5 M.?

4. How many inches in 1 Dm.? In 1 Hm.? In 1 Km.?

5. How many inches in 1 dm.? In 1 cm.? In 1 mm.?

6. Change 3,645 M. to Dm.; to dm.; to Hm.; to cm.; to Km.; to mm.

7. Change 8,716 dm. to m.; to Dm.; to cm.; to Hm.; to Km.

WRITTEN EXERCISES

6. Solve:

1. Express in one number:

(1) 15 Km. 9 Hm. 8 Dm. 5 m.

(2) 4 dm. 3 cm. 2 mm.

(3) 84 Km. 5 Hm. 6 Dm.

2. Express in units of each of the other denominations: 46,875.435 m.; 125,248.06 m.; 33,205.504 m.

3. Express the answers to the following in denominations of the Metric System:

(1) 486.5 m. × 25; by 2.5; by .25.

(2) 745.6 m. × $8\frac{1}{2}$; by $8\frac{3}{4}$; by $6\frac{5}{8}$.

(3) 375.5 m. ÷ 25; by 2.5; by .25.

4. A kilometer is about $\frac{5}{8}$ of a mile. Find the approximate number of miles in 15.5 Km.; in 36.05 Km.; in 88.75 Km.

5. Find the approximate number of kilometers in 15.9375 miles; in 31.875 miles; in 26.5625 miles.

6. Change the following to its approximate equal in our common system:

(1) 12 Km. 8 Hm. 6 Dm. 4 m.

SOLUTION: 12 Km. 8 Hm. 6 Dm. 4 m. = 12.864 Km.

12.864 × .625 = 7.7184 miles.

.7184 × 320 = 246.9888 rods.

.9888 rod × 16.5 = 14.3142 feet.

.3142 × 12 = 3.7704 inches.

8 miles, 246 rods, 14 feet, 3.7704 inches.

(2) 23 Km. 4 Hm. 3 Dm. 5 m.

(3) 34 Km. 7 Hm. 3 Dm.

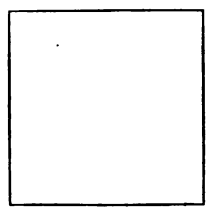
7. The length of a cable is 2 Km. 5 Hm. What is its approximate length in miles and rods?

AREA

7. The table of areas is formed by squaring the length measure, as in our common system.

For land measure 10 meters square is called an "**are**" (meaning "area"). The side of one are is about 33 feet. The **hectare** is 100 meters square, and, as its name indicates, is 100 ares, or about $2\frac{1}{2}$ acres. An acre is about 0.4 hectare. A standard United States quarter section contains almost 64 hectares. A square kilometer contains 100 hectares.

For smaller measures of surface the square meter is used. The square meter is about 20 per cent larger than the square yard. For still smaller surfaces the square centimeter is used. A square inch contains about $6\frac{1}{2}$ square centimeters.



Sq. In.

Sq. Cm.

8. METRIC SQUARE MEASURE

1 square kilometer (sq. Km.) = 1,000,000 sq. m.
1 square hectometer (sq. Hm.) = 10,000 sq. m.
1 square dekameter (sq. Dm.) = 100 sq. m.
1 square meter (sq. m.)
1 square decimeter (sq. dm.) = .01 sq. m.
1 square centimeter (sq. cm.) = .0001 sq. m.
1 square millimeter (sq. mm.) = .000001 sq. m.

1. How many sq. mm. in 1 sq. Km.?
2. How many sq. cm. in 1 sq. Hm.?
3. How many sq. dm. in 1 sq. Dm.?

Complete the table:

—— sq. mm. = 1 sq. ——
—— sq. cm. = 1 sq. ——
—— sq. dm. = 1 sq. ——
—— sq. m. = 1 sq. ——
—— sq. Dm. = 1 sq. ——
—— sq. Hm. = 1 sq. ——

By what ratio do the units of the denominations of this table increase and decrease?

9. ORAL EXERCISES

1. What are the dimensions in meters of a square dekameter?

2. A square dekameter equals how many square meters?

3. How many meters are there in the perimeter of a rectangle 3 dekameters long and 2 dekameters wide?

4. How many square meters are there in the rectangle? In a rectangle 8 dekameters long? 5 dekameters long?

5. What are the dimensions of a square hectometer in meters?

6. How many square meters in a square hectometer? How many square dekameters?

7. A rectangle whose sides are 4 hectometers and 3 hectometers, respectively, equals how many square dekameters? How many square meters?

8. What are the dimensions of a square kilometer in hectometers? In dekameters? In meters?

9. How many square hectometers in a square kilometer? How many square dekameters? How many square meters?

10. The side of a square decimeter equals what decimal of a meter?

11. A square decimeter equals what decimal of a square meter?

12. The side of a square centimeter equals what decimal of a meter?

13. A square centimeter equals what decimal of a meter?

14. A square millimeter equals what decimal of a meter?

15. How many square meters in 8 square kilometers? In 6 square hectometers? In 25 square dekameters? In 3 square decimeters? In 4 square dekameters, 8 square meters? In 5 square hectometers, 5 square dekameters?

10. WRITTEN EXERCISES

1. Express 5 sq. Hm. 18 sq. Dm. in square meters.

2. Express 25 Km. 15 Hm. 9 cm. in meters.

3. How many square meters in a rectangle 12 dm. long and 8 meters wide?

4. Mr. Milbrook has a yard 5 Hm. long and 18 Dm. wide, how many square meters in the yard?

5. Mr. Fairbank had a field 12 Hm. long and 25 Dm. wide; he sold 125,000 sq. m. How many square meters had he left?

11. METRIC LAND MEASURE

1 hectare (ha.) = 100 ares
1 are (a.)
1 centare (ca.) = .01 are.

The standard unit of metric land measure is an are, which is equivalent to 100 square meters, or approximately to .025 or $\frac{1}{40}$ of an acre.

1. What is the side of a square that contains 100 sq. m.?

2. Name a rectangle that contains 100 sq. m.

3. What is the approximate number of acres in 45 a.? In 86 a.? In 97 a.? In 36 a.? 25 ca.? In 53 a.? 17 ca.?

4. What is the approximate value of 15 ca. at $17 an acre? Of 265 a. at $50.75 an acre? Of 312 a. at $125.25 an acre?

5. Find the approximate value of 18 A. of land at $3.25 an a. Of 24 A. at $5.75 an a. Of 36 A. 80 sq. rd. at $4.80 an a. Of 25 A. 40 sq. rd. at $4\frac{1}{8}$ an a. Of 16 A. 32 sq. rd. at $3\frac{3}{4}$ an a.

NOTE. There are 40 times as many acres as ares, approximately.

6. Mr. Pierre has a tract of land 250 m. long 150 m. wide. What is its value at $650 an A.?

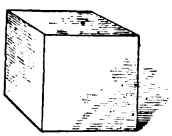
SOLUTION: $250 \times 150 = 37{,}500$ sq. m.;
$37{,}500$ sq. m. $\div 100 = 375$ a.;
375 a. $\times .025 = 9.375$ A.
$\$650 \times 9.375 = \$6{,}093.75$. *Ans.*

7. Find the approximate value of the following tracts of land:

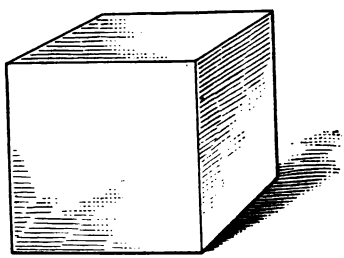
	LENGTH	WIDTH	PRICE PER A.
(1)	350 m.	210 m.	$400
(2)	175 "	140 "	$820
(3)	400 "	300 "	$260
(4)	550 "	400 "	$350
(5)	600 "	35 Dm.	$500
(6)	40 Dm.	30 Dm.	$525
(7)	80 rd.	60 rd.	$420
(8)	120 "	80 "	$500
(9)	96 "	60 "	$300
(10)	585 Dm.	170 Dm.	$100
(11)	210 m.	35 m.	$500
(12)	25 Hm.	13 Dm.	$400

VOLUME

12. The cubic measures are the cubes of the linear units. The cubic meter (sometimes called the **stere**, meaning "solid") is the unit of volume. A **cubic meter** of water weighs a **metric ton** and is equal to 1 **kiloliter**. The cubic meter is used in place of the cubic yard and is about 30 per cent larger. This is used for "cuts and fills" in grading land, in measuring timber, expressing contents of tanks and reservoirs, flow of rivers, dimensions of stone, tonnage of ships, and other places where the cubic yard and foot are used.



1 cu. cm.



1 cu. in.

The thousandth part of the cubic meter (1 cubic decimeter) is called the **liter**. (See table of capacity units).

For very small volumes the *cubic centimeter* (cu. cm. or $cm.^3$) is used. This volume of water weighs a *gram*, which is the unit of weight or mass. There are about 16 cubic centimeters in a cubic inch. The cubic centimeter is the unit of volume used by chemists as well as in pharmacy, medicine, surgery, and other technical work.

One thousand cubic centimeters make 1 liter.

1. Approximately 1 cu. ft. = how many $cm.^3$?
2. 1 cu. yd. = how many $cm.^3$?
3. 1 $m.^3$ = how many cu. ft.?
4. 1 cu. yd. = how many $m.^3$?

13. METRIC CUBIC MEASURE

1 cubic kilometer (cu. Km.)	1,000,000,000 cu. m.
1 cubic hectometer (cu. Hm.)	1,000,000 cu. m.
1 cubic decameter (cu. Dm.)	1,000 cu. m.
1 cubic meter (cu. m.)	
1 cubic decimeter (cu. dm.)	.001 cu. m.
1 cubic centimeter (cu. cm.)	.000001 cu. m.
1 cubic millimeter (cu. mm.)	.000000001 cu. m.

By what ratio do the units of this measure increase? Complete the table.

—— cu. mm. = 1 ——
—— cu. c. = 1 ——
—— cu. d. = 1 ——
—— cu. m. = 1 ——
—— cu. D. = 1 ——
—— cu. H. = 1 ——

NOTE. A cubic meter, or its equivalent, is called a stere (S.). A cubic decimeter is called a liter.

The standard unit of metric wood measure is a stere, which is $\frac{1}{4}$ of a cord, approximately.

14. METRIC WOOD MEASURE

1 dekastere (Ds.) = 10 steres.
1 stere (s.)
1 decistere (ds.) = .1 of a stere.

ORAL AND WRITTEN EXERCISES

15. Solve orally:

1. How many cubic decimeters in 5 cu. m.? In 7 cu. m.? In 9 cu. m.? In 4 s.? In 6 s.? In 8 s.? In 2 cu. m.? In 300 cu. dm.? In 15 cu. m. In 750 cu. Dm.?

2. How many cubic meters in 8,000 cu. dm.? In 35,000 cu. dm.? In 75,000 cu. dm.? How many steres in 4,000 cu. dm.? In 16,000 cu. dm.?

3. How many liters in 1 cu. dm.? In 5 cu. cm.? In 9 cu. cm.? In 500 cu. dm.? In 6 cu. cm.? In 250 cu. dm.? In 5 s.? In 3 s.? In 20 s.?

4. How many cords of wood in 15 s.? In 12 s.? In 10 s.?

5. A pile of wood 4 m. long, 3 m. wide, and 12 m. high contains how many steres? How many cords?

6. How many cords of tan bark in a pile 5 m. long, 4 m. wide, and 2 m. high? In a pile 6 m. long, 3 m. wide, and 2 m. high.?

16. Solve on paper:

1. Five boxes, each holding a stere, are full of packages of cereal. Each package holds a liter. What is the value of the cereal at 15 cents a package?

2. What the value of 2 steres of "Grape Nut," each package containing a liter, at the rate of two packages for 25 cents? Of $2\frac{1}{2}$ steres? Of 5.5 steres? Of 8.25 steres?

3. What is the approximate value of a pile of tan bark 8 m. long, 4 m. wide, and 2 m. high, at $6.25 a cord? Of a pile 10 m. long, 6 m. wide, and 4 M. high, at $5.75 a cord? At $7.50 a cord?

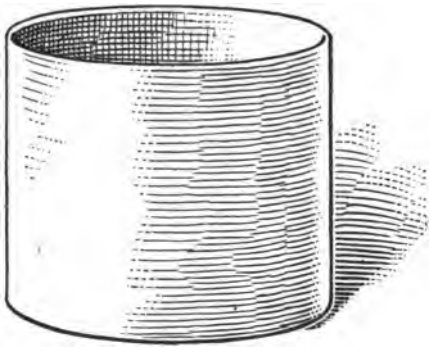
4. About how many cubic feet are there in a stere? In $7\frac{3}{4}$ s.? In 12.25 s.? In 15.5 Ds.? In 10.5 s.? In 17.25 s.? In 8.35 s?

5. What is the value of a pile of wood 5 Dm. long, 2 m. wide, and 2 m. high, at $3.98 a cord? Of a pile 4 Dm. long, 1 m. wide, and 2 m. high, at $6.70 a cord?

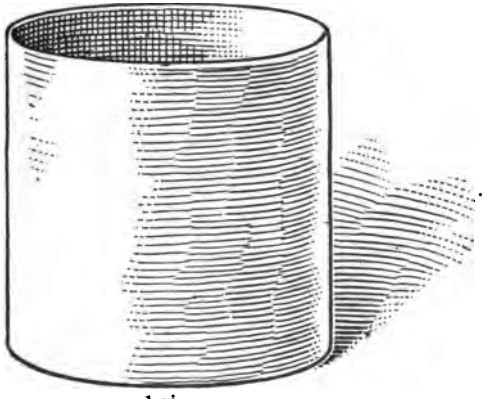
17. **UNITS OF CAPACITY**

milli-liter =	.001	liter,	ml.
centi-liter =	.01	"	cl.
deci-liter =	.1	"	dl.
liter =	1	"	l.
deka-liter =	10	"	Dl.
hecto-liter =	100	"	Hl.
kilo-liter =	1,000	"	Kl.

The *hectoliter* (100 liters) serves the same purpose as the United States bushel (2,150.42 cubic inches), and is equal to about 3 bushels, or a barrel. A peck is about 9 liters. The liter is used for measurements commonly given in the gallon, and the liquid and dry quarts, a liter being 5 per cent larger than our liquid



1 dl.



1 gi.

quart and 10 per cent smaller than the dry quart. A liter of water weighs exactly a kilogram, *i.e.* 1,000 grams. A thousand liters of water weigh 1 metric ton.

One liter equals 1.05658 liquid quarts or 0.9081 dry quarts.

One kilogram equals 2.204622 avoirdupois pounds.

ORAL EXERCISES

18. What is the ratio of:

1. 1 Kl. to 1 Hl.?
2. 1 Hl. to 1 Dl.?
3. 1 Dl to 1 l.?
4. 1 l. to 1 Dl.?
5. 1 Dl. to 1 cl.?
6. 1 cl. to 1 ml.?
7. 1 Kl. to 1 Dl.?
8. 1 Hl. to 1 l.?
9. 1 cl. to 1 l.?
10. 1 Ml. to 1 Dl.?
11. 1 Dl. to 1 Kl.?
12. 1 Ml. to 1 ml.?

WRITTEN EXERCISES

19. Solve as directed:

1. Express in one number:

4 Kl., 5 Hl., 2 Dl., 6 l., 3 dl., 7 cl., 8 ml.

2. Express 6275.325 l. in units of each of the other denominations.

3. Write and add:

8 Kl., 0 Hl., 5 l., 3 dl., 0 cl., 4 ml., 3 Kl., 6 Hl., 9 Dl., 2 l., 0 dl., 4 cl., 5 Kl., 7 Hl., 0 dl., 3 l., 5 dl., 0 cl., 5 ml.

4. From 8,645.5 l. take 5,468.15 l.

5. 575.25 l. $\div$ 5.

20. The liter is equal to about .9 of a quart dry measure and 1.05 quarts liquid measure.

How many quarts dry measure in 4 l.?

Since there are .9 quart in 1 l., there are in 4 l. .9 quart $\times$ 4 = 3.6 quarts.

ORAL EXERCISES

21. How many quarts dry measure are there in:

6 l.? 8 l.? 11 l.? 12 l.? 9 l.? 5 l.? 1 Dl.? 2 Dl.? 5 Dl.? 8 Dl.? 3 Dl.? 7 Dl.? 1 Hl.? 5 Hl.? 4 Hl.? 9 Hl.? 6 Hl.? 3 Hl.? 1 Kl.? 5 Kl.? 4 Kl.? 8 Kl.? 9 Kl.?

WRITTEN EXERCISES

22. **1.** Find the number of quarts, liquid measure, that each one of the above metric expressions equals.

2. Mr. Winthrop sold 4 Dl., 4 l. of syrup. What is its value at $.18 per quart?

SOLUTION: 5 Dl., 4 l. = 54 L.; 1.05 quarts × 54 = 56.7 quarts.
$.18 × 56.7 = $10.206.

3. What is the value of 6 Dl., 5 l. of alcohol at $.80 per quart?

4. What are 2 Dl., 3 l. of chestnuts worth, at $.12 per quart?

5. What are 1 Hl., 5 Dl. of beans worth, at $.15 per quart?

6. A grocer sold 4 Hl., 4 Dl. of vinegar at $.32 per gallon. How much did he receive?

7. A vendor sold 5 Hl., 6 Dl. of peanuts at $.80 a peck. What did he receive for them?

8. Change 3 bushels, 1 peck, 5 quarts, 1 pint to liters.

SOLUTION: 3 bushels, 1 peck, 5 quarts, 1 pint = 109.5 quarts.
109.5 ÷ .9 = 121.66 liters.

9. How many liters are there in 28 bushels, 2 pecks, 4 quarts, 1 pint?

10. How many liters are there in 16 bushels, 3 pecks, 3 quarts, 1 pint?

11. How many liters are there in 43 bushels, 1 peck, 7 quarts, 1 pint?

12. How many liters are there in 12 gallons, 3 quarts, 1 pint, 3 gills?

SOLUTION: 12 gallons, 3 quarts, 1 pint, 3 gills = 51.875 quarts.
51.875 ÷ 1.05 = 49.404 liters.

13. How many liters are there in 85 gallons, 2 quarts, 1 pint, 2 gills?

14. How many liters are there in 64 gallons, 3 quarts, 0 pints, 3 gills?

15. How many liters are there in 15 gallons, 0 quarts, 1 pint, 1 gill?

16. A grocer bought 20 bushels, 3 pecks, 6 quarts of hickory nuts for 8 cents a quart and sold them for 8 cents a liter? How much did he make?

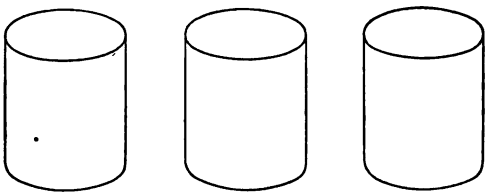
17. If 80 gallons, 3 quarts of molasses were bought at 16 cents a quart and sold at 20 cents per liter, how much would be made?

23. UNITS OF WEIGHT (OR MASS)

milli-gram =	.001	gram,	mg.
centi-gram =	.01	"	cg.
deci-gram =	.1	"	dg.
gram =	1	"	g.
deka-gram =	10	"	Dg.
hecto-gram =	100	"	Hg.
kilo-gram =	1,000	"	Kg.

Measurements commonly expressed in gross tons or short tons are stated in metric tons (1,000 kilograms). The metric ton comes between our long and short tons and serves the purposes of both.

The kilogram and "half kilo" serve for everyday trade, the latter being 10 per cent larger than the pound. The kilogram is approximately 2.2 pounds.



Relative size of avoirdupois ounce, 30-gram, and Troy ounce (brass) weights. (Actual size.)

The gram and its multiples and divisions are used for the same purpose as ounces, pennyweights, drams, scruples, and grains. For foreign postage, 30 grams is the legal equivalent of the avoirdupois ounce.

Gram Scruple

Relative size of gram and scruple (brass) weights. (Actual size.)

Complete the table:

—— mg. = 1 ——
—— cg. = 1 ——
—— dg. = 1 ——
—— g. = 1 ——
—— Dg. = 1 ——
—— Hg. = 1 ——

A kilogram is equal to about $2\frac{1}{5}$ lbs.

About what decimal of a pound does a Hg. equal? A Dg.? A gram?

ORAL EXERCISES

24. Solve:

1. How many grams in 4 Dg.? In 6 Dg.? In 3 Hg.? In 5 Hg.? In 7 Hg.? In 7 Kg.? In $9\frac{1}{2}$ Kg.? In 15 Dg.? In 12 Hg. 4 Dg.? In 16 Kg. 8 Hg. 5 Dg.?

2. What decimal of a gram is 6 dg.? 8 dg.? 5 cg.? 8 cg.? 3 mg.? 9 mg.?

3. Express in grams: 5 g. 5 dg.; 8 g. 6 cg.; 12 g. 14 mg.; 25 g. 2 dg.; 3 cg. 5 mg.; 4 dg.; 5 g. 3 dg. 7 cg. 5 mg.

4. About how many pounds do 4 Kg. equal? 6 Kg.? 10 Kg.? 20 Kg.? 5 Hg.? 11 Hg.? 12 Hg.? 15 Hg.? 25 Hg.? 5 Dg.? 12 Dg.? 15 Dg.? 10 g.? 100 g.? 1,000 g.? 4,000 g.? 5,000 g.?

WRITTEN EXERCISES

25. Solve:

1. What is the approximate value of 25 kilograms of coffee at 32 cents a pound? Of 36.5 kilograms at 28 cents a pound? Of 18.75 kilograms of tea at 48 cents a pound?

2. Mr. Anthony bought 46.75 kilograms of cheese at $24\frac{1}{2}$ cents a kilogram, and sold it at 15 cents per pound. How much did he make?

3. Abouthow many kilograms in 22,000 lb.? In 55,000 lb.? In 401,500 lb.?

4. The Ansonia Company bought 132,000 lb. of copper at 13 cents a pound and sold it in Paris at 33 cents a kilogram. What was the gain?

5. What is the value of 5,250 g. of butter, at 56 cents a kilogram?

6. What is the value of 520 g. of cheese at $.52 a kilogram?

7. What is the cost of 6,500 Kg. of coal at $6.50 a ton? At $5.75 a ton?

8. What will 50 Kg. of tea cost at $.60 a pound? 40 Kg. at $.50 a pound?

A TABLE OF EQUIVALENTS

26. A liter equals about .9 qt. dry measure.
A liter equals about 1.05 qt. liquid measure.
A meter equals 39.37 inches linear measure.
A kilometer equals about .625, or $\frac{5}{8}$ mile, linear measure.
An are equals about .025, or $\frac{1}{40}$ acre, square measure.
A stere equals about .25, or $\frac{1}{4}$ cord, of wood.
A gram equals about .0025, or $\frac{1}{400}$ lb., avoirdupois.

1. Louis sold 12 bushels, 3 pecks, 6 quarts of hickory nuts, at 8 cents a liter. How much did he receive for them?

2. How much are 2,736 liters of cranberries worth, at 12 cents a quart?

3. Mr. France sold 25 gallons, 3 quarts, 1 pint of vinegar, at 6 cents a liter. How much did he receive for it?

4. What is the value of 1,008 liters of molasses, at 16 cents a quart?

5. How many inches in 5 Dm. 4 m.? How many feet?

6. What will it cost to macadamize a road 24.16 kilometers long, at $3,250 a mile?

7. What will it cost to build a Telford road 5.5 miles, at $3,260 a kilometer?

8. What is the approximate value of 2,880 acres of land, at $87.50 an acre?

9. What is the approximate value of 8.87 acres, at $4.25 an acre?

10. A grocer sold 68,000 grams of sugar, at $4\frac{1}{8}$ cents a pound. About how much did he receive?

11. A package of silver weighs 3.47 grams. What is its weight in grains?

12. How many 2-gr. capsules will 5 g. of quinine fill?

CASTING OUT THE NINES

Very sure tests of the four fundamental processes are obtained by "casting out the nines"; that is, subtracting 9 as many times as possible from the sum of the digits.

Cast out the nines from 68,574.

Beginning at the left, add the digits until their sum equals or exceeds 9: $6+8=14$

Subtract 9: $14-9=5$

Continue the addition, beginning with the difference. $5+5=10$

Subtract 9. $10-9=1$

Continue the addition. $1+7+4=12$

Subtract 9. $12-9=3$

3, the last difference, is called the **excess**.

To test addition by casting out the nines. Cast out the nines from each addend and from the sum of the excesses if this sum equals or exceeds 9. Then cast out the nines from the sum. If the excess of the addends equals that of the sum, the addition is correct.

Add and test:

68,574	Excess 3	$3+8=11$
43,928	Excess 8	$11-9=2$
112,502	Excess 2	Excess of excesses 2.

As the excess of the excesses of the addends equals the excess of the sum, the result is correct.

Add and test:

1.	2.	3.
15,984	111,999	400,004
36,725	222,888	500,005

To test subtraction by casting out the nines. Cast out the nines of the minuend and the subtrahend and the difference. If the excess of the minuend equals the excess of the sum of the excesses of the difference and subtrahend, the subtraction is correct.

Subtract:	75,864	Excess 3	$5+7=12$
	58,912	Excess 7	Excess 3
	16,952	5	

Subtract and test:

1.	2.	3.	4.
19,873	98,764	100,000	44,444
14,569	10,801	98,705	19,999

To test multiplication by casting out the nines. If the excess of the product equals the excess of the product of the excesses of multiplicand and multiplier, the multiplication is correct.

485	Excess 8	$8 \times 5 = 40$	Excess 4
23	Excess 5		
1455			
970			
11155	Excess 4		

Multiply and test:

1.	2.	3.	4.
6783	5467	383	68,573
98	76	383	284

To test division by casting out the nines. Multiply the excess of the quotient by that of the divisor. If the excess of the result equals that of the dividend, the division is correct.

If there is a remainder, add the excess of the product of the divisor times the quotient to the excess of the remainder and find the excess of this sum. This should equal the excess of the dividend.

Divide 6754 by 32.

211		
32)6754	Excess of dividend	4
64	Excess of divisor	5
35	Excess of quotient	4
32	Excess of remainder	2
34		
32	$5 \times 4 = 20$	Excess 2
2	$2 \times 2 = 4$	Excess of dividend.

Divide and test:

1. 33)9999; 41)8576; 57)7384; 902)876,549.

TABLES OF DENOMINATE NUMBERS

(1)

MEASURES OF LENGTH

2 inches	= 1 foot
3 feet	= 1 yard
16½ feet	= 1 rod
320 rods	= 1 mile

1 mi. = 1760 yd. = 5280 ft. = 63,360 in.

A *nautical mile* (*knot*) = 6080.27 ft., or approximately 1.15 mi.

A *furlong* = ⅛ mi.; a *fathom*, used in measuring the depth of water, is 6 ft.; a *hand*, used in measuring the height of horses, is 4 in.

(2)

SQUARE MEASURES

144 square inches	= 1 square foot
9 square inches	= 1 square yard
30¼ square yards	= 1 square rod
160 square rods	= 1 acre
1 acre	= 43,560 sq. ft.

An acre of land in the form of a square is very nearly 209 feet on a side.

A tract of land 1 mile square is often called a section.

100 sq. ft. of roofing, flooring, or slating is called a square.

(3)

CUBIC MEASURES

1728 cubic inches	= 1 cubic foot
27 cubic feet	= 1 cubic yard

A pile of wood 8 ft. long, 4 ft. wide, and 4 ft. high, or 128 cu. ft. of wood, is called a cord.

(4)

SURVEYORS' LINEAR MEASURES

100 links = 1 chain
80 chains = 1 mile

This chain, called *Gunter's chain*, is 4 rd., or 66 ft., long. 1 *link* = 7.92 in.

Links are written as hundredths of a chain.

(5)

SURVEYORS' SQUARE MEASURE

10 square chains = 1 acre
640 acres = 1 square mile

(6)

LIQUID MEASURES

4 gills = 1 pint
2 pints = 1 quart
4 quarts = 1 gallon

1 gal. = 231 cu. in.; 1 cu. ft. = $7\frac{1}{2}$ gal., approximately. A gallon of water weighs about $8\frac{1}{3}$ lb.; a cubic foot of water weighs about $62\frac{1}{2}$ lb., or 1000 oz.

In measuring the capacity of cisterns, etc., $31\frac{1}{2}$ gal. = 1 *barrel.*

(7)

DRY MEASURES

2 pints = 1 quart
8 quart = 1 peck
4 pecks = 1 bushel

1 bu. = 2150.42 cu. in., or approximately $1\frac{1}{4}$ cu. ft.

Our bushel is the *Winchester bushel.* In form it is a cylinder $18\frac{1}{2}$ in. in diameter and 8 in. deep.

(8)

AVOIRDUPOIS WEIGHT

16 ounces = 1 pound
100 pounds = 1 hundredweight
2000 pounds = 1 ton
1 *long* or *gross* ton = 2240 pounds

1 av. lb. = 7000 gr.; 1 av. oz. = $437\frac{1}{2}$ gr.

(9)

TROY WEIGHT

24 grains = 1 pennyweight
20 pennyweights = 1 ounce
12 ounces = 1 pound

1 troy pound = 5760 gr. = $\frac{5760}{7000}$ av. lb.
1 troy ounce = 480 gr., or about 1.1 av. oz.
Troy weight is used in weighing gold, silver, and other precious metals and gems.

(10)

APOTHECARIES' WEIGHT

Apothecaries' measures are used to some extent by druggists in filling prescriptions. The grain, ounce, and pound are the same as those in Troy weight, but the ounce is divided differently.

20 grains (gr.) = 1 scruple . . . sc. or ℈.
3 scruples = 1 dram . . . . dr. or ʒ.
8 drams = 1 ounce . . . oz. or ℥.
12 ounces = 1 pound . . . lb. or ℔.

The grain is the basis of our three systems of weights. As its name implies, it was originally the weight of a single grain of wheat.

(11)

APOTHECARIES' LIQUID MEASURES

60 drops (gtt.) or minims (m.) = 1 fluid dram . $f\mathfrak{z}$.
8 fluid drams = 1 fluid ounce . $f\mathfrak{z}$.
16 fluid ounces = 1 pint . . . O.
8 pints = 1 gallon . . . Cong.

(12)

MEASURES OF TIME

60 seconds = 1 minute
60 minutes = 1 hour
24 hours = 1 day
7 days = 1 week
365 days = 1 year
366 days = 1 leap year

10 years = 1 *decade*. 100 years = 1 *century*.

Thirty days have September,
April, June and November.
All the rest have thirty-one,
Save February, which alone
Has twenty-eight, and one day more
We add to it one year in four.

The earth revolves around the sun in 365 days, 5 hours, 48 minutes, 46 seconds. This is the *mean solar year*, and is nearly $365\frac{1}{4}$ days.

To correct the errors in the calendar, made by disregarding the fraction of a day over 365 days, centennial years divisible by 400 and other years divisible by 4 are lengthened 1 day, Feb. 29. These years are *leap years*.

The standard unit of time is the *day* ("the mean solar day," which is the average time between the noon of one day and noon of the day following).

(13)

COUNTING TABLE

2	= 1 pair
20	= 1 score
12	= 1 dozen
12 dozen	= 1 gross
12 gross	= 1 great gross

(14)

STATIONERS' MEASURES

24 sheets	= 1 quire
20 quires	= 1 ream
20 reams	= 1 bundle
5 bundles	= 1 bale

Paper is quite generally sold by the 100, 500, and 1000 sheets; also by the pound.